Impossible Probability:

Anthology,

Vol 1.

I doubt, therefore I exist.

Dr Ruben Garcia Pedraza

Preliminary Words for the Publication of *Impossible Probability, Anthology*, in Spanish:

Impossible Probability

The work presented here is a compilation of the posts I published on the blog *Impossible Probability* between 1 August 2011 and 13 January 2018, the posts I wrote in Spanish. From 13 January onwards, while living and working in London in different schools, and given the importance we already knew Global Artificial Intelligence was going to have, I would try to write them in English with the help of online correctors and translators, especially Google, and later I would use Grammarly.

The originals written in English, with all their imperfections, are still preserved on the *Impossible Probability* blog, and also on Internet Archive, where my poetic works can also be found.

As I have said on numerous occasions, especially in my poetry, behind every imperfection, behind every subtlety, entire hidden worlds are concealed, full of mystery and uncertainty yet to be discovered, and that the adventurous and hungry mind can find as easily as going to Internet Archive, where all my secrets await the right reader who knows how to translate them—like someone who by mistake finds a secret entrance, the passage leading to the cave where the elves who guard hidden knowledge dwell, and the dragons who defend the golden fortress.

While the original texts in English can be found on Internet Archive, the corrected and expanded version, this time using ChatGPT, can be found in scientific repositories (Zenodo, ORCID, PhilPapers) and on Amazon. In any case, if in the future a comparative study were carried out between my original texts in the *Impossible Probability* blog and Internet Archive, and the corrected and expanded versions, it would be clear without any difficulty that the original contents are perfectly preserved, and the corrected and

expanded versions are faithful to the contents we wrote together in 2018 in relation to Gaia, Mother, the Global Artificial Intelligence.

The importance of the documents that in this new series of books *Impossible Probability: Anthology* will be published in Spanish, containing the articles between 1 August 2011 and 13 January 2018, lies in the fact that we began to write this blog just a few weeks before the death of José Rodríguez Delgado, and a few months before the publication of our book *Introduction to Impossible Probability, Statistics of Probability and Statistical Probability.*

Although it is very difficult at the moment to explain who "we" are, and what *Impossible Probability* means for us, what matters is the work we leave: our work, our legacy, our *legend*, *our symbol*, our seal—independently of the judgment that history may later make of our contribution to the development of mathematics, Artificial Intelligence, and most importantly, Cyborg Psychology.

The first time we began to write about *Impossible Probability* I was only a student in the Faculty of Education at the Complutense University of Madrid [the Dean was Gonzalo Vazquez]. In my first year of university I had as professor Jesús Martín Ramírez, collaborator of Karl Pribram and author of the brain hologram theory, which is the origin of cyborg psychology. While José Delgado provided the structure, the stimoceiver, nevertheless, to understand the functioning of this technology in the 1970s and 80s, Pribram's contribution was fundamental—and even today it still is.

My first year of university was fabulous: good grades, good vacations hiking, summer camps in the mountains of Madrid—and suddenly, in the second year of university 1996–1997, my life would change abruptly, which can be seen in my poetry, how my poems began to acquire a more dramatic tone.

I repeated statistics in my second year, I repeated it at least three years, and I was even on the verge of being expelled from university. Miraculously, it did not happen, and precisely the last year I had left to pass statistics, I passed. The following year I would begin to write *Impossible Probability* as a critique of Karl Popper—and in a certain sense, it is.

Whereas for Karl Popper knowledge is false, only accepted provisionally as true until we refute it, in *Impossible Probability* rational knowledge is provisionally true while we investigate true pure knowledge—in the ultimate instance, our search for knowledge of the noumenon, which the human being cannot reach, hence the necessity of Global Artificial Intelligence: that machine capable of knowing what happens beyond human limitations.

What we call chaos, error, complexity, is only the noise produced by our human cognitive limitations, given our apriorism. The human being, limited both sensorially and cognitively, cannot know what really happens, but we can build the machine that can reach true knowledge of what happens—the truth in itself and for itself. That is, and no other, the true end of Artificial Intelligence: to reach the knowledge of truth, whatever it may be.

The path that takes me from my first works in *Impossible Probability* to Global Artificial Intelligence is not easy: much suffering, even despair, sometimes a sense of helplessness, a sense of abandonment—yet an inner force always pushed me forward. That inner force is what I call "we," although I understand that at this moment it is inexplicable.

Although I feel a bond that unites me to the *legend* of José Rodríguez Delgado, and which my Stockholm syndrome prevents me from understanding, my bond with Pribram is evident: Jesús Martín Ramírez. Pribram and Delgado worked at Yale University; probably only the skulls can say whether both were members of Skull & Bones, though there is no evidence to corroborate this.

Impossible Probability is a work that comes out of the romanticism of a student poet, who believed in white fairies, the rays of Zeus, and the light of the god Apollo illuminating the heart of every spirit and soul in a world full of mysteries and shadows.

It was from that world of poetry, in a country emerging from a failed transition, that the last survivor of the dream of reason would listen to his inner voice, dark and serene, and *Impossible Probability* would be born for the first time.

The first formulations of *Impossible Probability* date from 11 September 2001. My first formula would be the Impact of the Defect; then on 16 October 2002 the Second Method would appear, laying the foundations of the distinction between empirical and theoretical probability. I worked intensely until 2004. Between 2004 and 19 December 2009, I paused my mathematical frenzy, and it was not until 19 December 2009 when I returned, this time with the intention of publishing *Introduction to Impossible Probability, Statistics of Probability or Statistical Probability*, whose publication materialized just a few months after the death of José Rodríguez Delgado, although Delgado was able to see the first works published online in this *Impossible Probability* blog, which makes this blog a fundamental piece for understanding our work, our legacy, and most importantly—independently of our names, which matter little—the truly essential: our seed of the future, our eternal gratitude, a work of intelligence that will lay the foundations of the future technological society.

Dr. Rubén García Pedraza

20 August 2025, 19:00

Madrid, Villaverde

Preliminary Words for the English Version of *Impossible Probability, Anthology*.

Spanish-speaking readers have been able to enjoy our work since we began the foundational work of *Impossible Probability* in 2001, more precisely when we began to make it public in July 2011 through the *Impossible Probability* blog. The work presented here is an extended version of the posts published on the blog, an extended version translated into English with the help of Gepeto, my special GPT program.

In any case, a comparative study of the augmented translation presented here and the original works in Spanish could corroborate that the essential contents related to formulation and the philosophy of mathematics remain in accordance with the original electrical spirit of *Impossible Probability*.

The reason that obliges me to publish so hastily the English-language publication of all my works is the very complicated personal situation in which I find myself, due to serious mistakes committed by certain British institutions, which, completely blind to what our situation is, instead of having supported our program from the first moment, tried to sabotage it from the beginning through numerous obstacles and false loves.

I have no doubt that the manipulation to which I have been subjected by British institutions will one day be investigated, and at some point justice must make an appearance.

When that day comes, the first thing justice must recognize is the extreme difficulty and harshness of the conditions I have lived all my life since my earliest childhood. From the very day I came out of my mother's womb in Spain I have known nothing but pain and the oblivion of a society that prefers to look the other way and wash its hands, pretending that nothing is happening here. In the midst of the dictatorship of silence, I have had to survive all kinds of traps, deceptions, and attempts at assassination. The dictatorship of silence, the Roman circus, and the virtual zoo that my life has become will one day have to end. For this reason I beg the International Court of Justice that when that day comes the matter be investigated down to the smallest detail, even being

aware that in 48 years of savage dictatorship of silence I have developed Stockholm Syndrome. Under the harsh conditions in which I have had to learn to survive, if I have learned anything it is the importance of survival. I beg the International Court of Justice to understand what it means to survive 48 years of dictatorship living in a Roman circus full of traps and lions hidden in every corner.

I deeply regret having to publish my mathematical works under these conditions, and in record time doing all the translations I can, but at this moment I can only say that my life is at risk.

If one day I could testify before the International Court of Justice I beg you to understand the brutal complexity of the situation in which I have lived since the first minute I came out of my mother's womb. I have learned to survive.

Dr Ruben Garcia Pedraza

16 September 2025, 12:58

London, Leytostone

“La serie aleatoria de incoherencias sin lógica

Que en si mismo tiene sentido o

Forma de mensaje oculto”

“The random series of incoherences without logic

That in itself has meaning or

Form of a hidden message.”

Poem *Matematicas*

Ruben Garcia Pedraza

Written at Royal Albert Hall

London

Southbank

Impossible Probability: Anthology, Vol 1.

Dr Ruben Garcia Pedraza

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1. Empirical Probability and Theoretical Probability

1.1. Origins and Initial Motivations

The theory of *Impossible Probability* is a model of statistical thought that first emerged in the spring of 2001. Before the development of this theoretical framework, my intellectual and creative inclinations were oriented primarily toward poetry, theatre, and a certain form of romantic philosophy. These earlier pursuits reflected a search for beauty, imagination, and meaning in the symbolic and artistic dimensions of human life. They were essential in shaping my sensibility but remained, in many respects, distant from the mathematical and scientific rigor that would later define the construction of *Impossible Probability*.

It was precisely during the spring of 2001 that I began to experience a decisive intellectual shift. *I felt an urgent need* to move beyond the purely aesthetic and literary domains in order to engage with a philosophical and mathematical model that could represent a significant step forward in positive and constructive thought. This *transition* did not imply abandoning poetry or theatre; rather, it meant complementing them with a scientific orientation that could address the fundamental question of how reality organizes itself, and how such organization can be understood through probability and statistics.

The motivation for this transition lay in the *observation* of an apparent contradiction between the theoretical formulations of classical probability and the empirical manifestations of reality. Traditional probability theory posits that an event whose probability is equal to zero is considered impossible. However, in the lived world, **events that should theoretically never occur according to this definition do, in fact, occur on a daily basis**. This contradiction between the theoretical "impossibility" of

certain events and their empirical actualization became the key intuition that demanded a new conceptual model.

Thus, *Impossible Probability* arose as a philosophical and scientific response to this paradox. It was an attempt to reconcile the formal rigidity of mathematical probability with the openness and unpredictability of real-world phenomena. The project sought to develop a theoretical model capable of explaining how events deemed impossible within classical probability frameworks could nevertheless manifest in empirical reality. The result was the beginning of a new dialectical approach: to treat probability not only as a numerical expression of likelihood but also as a critical tool for understanding the contradictions inherent in reality itself.

In this sense, the initial motivations behind *Impossible Probability* were not merely mathematical but profoundly philosophical. The theory was conceived as part of a broader intellectual *journey*, moving from the romantic exploration of meaning in literature and philosophy toward a rigorous yet critical engagement with science. The spring of 2001 therefore represents the point at which a poetic sensibility merged with mathematical inquiry, laying the groundwork for a model that aspires to redefine the relationship between probability, impossibility, and the empirical structure of the world.

1.2. The Fundamental Intuition: The Impossible That Happens

At its core, *Impossible Probability* designates any probability that is mathematically defined as zero and therefore theoretically described as “impossible.” According to classical statistical logic, an event whose probability equals zero should never occur, for it has been excluded by definition from the domain of possible outcomes. This formal treatment of zero-probability events is one of the cornerstones of traditional probability theory, where impossibility is not merely an empirical claim but a logical necessity derived from the axioms of the discipline.

The central intuition that began to occupy my thought in the spring of 2001 arose precisely in the tension between this rigid theoretical framework and the empirical evidence offered by daily experience. If the theory states that zero probability implies

impossibility, then reality itself seems to resist such an assertion. **In lived experience, one repeatedly encounters events that, under the framework of traditional probability, should never occur**. These events, although unlikely to the point of being theoretically impossible, nonetheless manifest themselves within the unfolding of the real world.

This paradox—that the "impossible" not only occurs but occurs with a frequency sufficient to be noticed—provided the initial impetus for the creation of *Impossible Probability*. What intrigued me was not simply the existence of rare or extraordinary events, but the fact that their appearance contradicted the absolute exclusion implied by the concept of zero probability. This contradiction suggested that probability, as traditionally understood, might not be an exhaustive tool for comprehending the structure of reality. Instead, probability seemed to conceal within itself a dialectic: on the one hand, it defined rigid mathematical boundaries; on the other, reality demonstrated the porousness of those very boundaries.

To grapple with this contradiction, *Impossible Probability* proposed a rethinking of how one interprets events of zero probability. Rather than dismissing them as excluded from reality, the theory recognized that their very occurrence highlights the limitations of existing probabilistic models. In this sense, the "impossible that happens" becomes the object of study: it is not a statistical anomaly to be disregarded, but a necessary dimension of reality that requires its own formal explanation.

This fundamental intuition transformed the apparent impossibility of such events into a central problem of scientific investigation. By taking seriously the contradiction between theoretical impossibility and empirical occurrence, *Impossible Probability* began to *chart a path toward a critical theory of probability*—one that does not merely describe frequencies or likelihoods but seeks to integrate the contradictions of reality itself into a coherent mathematical and philosophical framework.

1.3. Historical Development of the Theory

The theoretical construction of *Impossible Probability* unfolded in two distinct phases, each marked by different rhythms of development and critical milestones. The first phase extended from the spring of 2001 until the summer of 2004, although in 2005 I continued to draft a number of mathematical developments that remained at the stage of sketches and preliminary outlines. These early years constituted the formative period of the theory, a time when intuition and philosophical reflection were gradually shaped into a coherent mathematical framework.

Within this first phase, one date stands out as a turning point: October 16, 2002. On this day I formulated what would later be known as the "Second Method," a conceptual breakthrough that decisively marked the trajectory of *Impossible Probability*. The Second Method was not a minor adjustment or a reinterpretation of existing probability theory; rather, it introduced an entirely new distinction between empirical probability and theoretical probability. Unlike the conventional definitions that dominate classical statistics, this distinction opened a new pathway for the statistical study of reality by proposing that probabilities could be understood in terms of their empirical manifestations while simultaneously being compared against their theoretical expectations.

The introduction of the Second Method had profound implications. It suggested that probability could no longer be confined to the study of relative frequencies or direct scores in isolation but had to be treated as a dialectical relationship between what occurs in practice (empirical probability) and what should occur in principle (theoretical probability). By formally articulating this distinction, the theory provided the mathematical tools needed to address the paradox introduced in the initial intuition: the occurrence of the impossible.

The second phase of development began on December 19, 2009, and extended into the years leading up to the publication of *Introduction to Impossible Probability: The Statistics of Probability or Probabilistic Statistics*. This phase was characterized by a systematic effort to consolidate, refine, and present the theory in a more complete and

rigorous form. Whereas the first phase was exploratory and marked by bursts of innovation, the second was more deliberate, involving the task of integrating the insights of the Second Method into a broader framework that could sustain academic scrutiny and practical application.

Taken together, these two phases represent the intellectual journey of *Impossible Probability* from its origin in a moment of philosophical intuition to its establishment as a formal theory. The first phase provided the foundation by defining its central concepts and methods, while the second phase marked the beginning of its dissemination and maturation as a contribution to statistical thought. In this historical progression, the theory demonstrates not only its mathematical ambition but also its capacity to evolve, to refine itself, and to respond to the demands of a scientific model that seeks to account for the contradictions of reality.

[This augmented translation corresponds to the first post in the blog Probabilidad Imposible, the blog itself till Christmas 2027 could be considered the third phase of Impossible Probability, our research on GAI since Christmas 2017 till 2019 the fourth phase of Impossible Probability, what means that now we are in the fifth phase. Actually, the current mission of Impossible Probability is to set up the international legal framework for cyborg robotics at the International Court of Justice, it is not coincidental that the fifth phase in GAI framework is the development of particular programs]

1.4. The Second Method and Its Significance

The importance of October 16, 2002, lies in the formulation of what came to be known as the *Second Method*. This methodological innovation represented a decisive departure from the established practices of classical statistics. Traditionally, statistical theory maintains a clear distinction between the domains of statistics and probability: on the one hand, statistics concerns itself with the analysis of direct scores—using concepts such as the arithmetic mean, mean deviation, or standard deviation to describe the distribution of observed values; on the other hand, probability is generally reserved for the study of frequencies or relative likelihoods, abstracted from the actual values of the data.

The Second Method challenged this conventional boundary. By rethinking the relationship between direct scores and probabilities, it proposed a framework in which probability itself could be studied statistically, and statistics could be extended into the domain of probability. In other words, the method enabled the development of statistical analyses of probability itself, rather than limiting probability to an auxiliary role or a theoretical background for the study of frequencies. This was not simply a technical modification but a conceptual reorientation: the Second Method made it possible to treat the same dataset simultaneously as a collection of direct scores and as a field of probabilistic relations.

The main difference introduced by the Second Method, when contrasted with classical or traditional statistical approaches, was its capacity to generate probability studies over any possible universe of data. This universality marked a profound expansion of the scope of statistical inquiry. Whereas traditional models typically restricted probability to the study of relative frequencies within a sample, the Second Method allowed for the treatment of both frequencies and direct scores as interchangeable perspectives within a unified framework.

To illustrate this innovation, consider two types of analyses: the first involves examining the relative frequencies of different options within a dataset; the second involves analyzing the direct scores obtained by individual subjects within a sample of N subjects. Under traditional statistics, these two analyses would belong to different methodological domains. The Second Method, however, renders them commensurable: frequencies can be treated as if they were direct scores, and direct scores can be studied as if they were probabilities. The distinction between the statistical and the probabilistic dissolves, giving rise to a single model capable of encompassing both.

The transcendence of this shift lies in its implications for the philosophy of science as well as for mathematical practice. By refusing to accept the rigid partition between statistics and probability, the Second Method redefines the way in which empirical data can be interpreted. It expands the conceptual tools available for the analysis of reality, making it possible to construct models that capture not only the distribution of observed data but also the underlying probabilistic structures that govern their

manifestation. In doing so, it provides the foundation for *Impossible Probability* as a critical theory—one that situates the paradox of the impossible event within a coherent mathematical framework, while simultaneously broadening the horizons of statistical reasoning beyond its classical limits.

1.5. Definition of Empirical Probability

In order to make effective the mathematical synthesis proposed by the Second Method, it was necessary to introduce a concept that could serve as the cornerstone of the new statistical model: *empirical probability*. This notion was designed to enable the treatment of direct scores as if they were frequencies, and of frequencies as if they were direct scores, thereby erasing the rigid separation that had previously divided these two domains in classical statistical analysis.

Empirical probability in the Second Method is defined as the ratio of the direct score or frequency of a given subject or option to the sum of all direct scores or frequencies within the sample. In other words, it expresses the proportional *contribution* of an individual element—whether a subject's measured score or the frequency of an option—to the totality of values observed in the dataset. By grounding probability in this proportional relation, the Second Method provided a way of unifying different statistical perspectives under a single formula.

Formally, empirical probability is expressed as follows:

$$p(x_i) = \frac{x_i}{\sum x_i}$$

Where:

- $p(x_i)$ = empirical probability of a subject or option
- x_i = the direct score of a subject or the frequency of an option
- $\sum x_i$ = the sum of all direct scores or frequencies in the sample

This definition is deceptively simple, but it *contains* profound implications. It asserts that every subject or option in a sample can be assigned a probability that reflects its relative weight within the whole. Thus, a direct score that might traditionally have been

treated only as raw data is reconceived as a probability measure, and likewise, a frequency that might traditionally have been seen only as a relative occurrence is treated as a direct contribution to the total.

Through this formulation, the concept of empirical probability becomes the foundational link that allows the Second Method to bridge the gap between classical statistics and probability theory. It transforms the structure of analysis by making it possible to study scores and frequencies through the same mathematical lens, thereby creating the *conditions* for a critical reinterpretation of probability. This redefinition does not eliminate the traditional concepts of mean, variance, or frequency; rather, it integrates them into a broader framework that *emphasizes* proportionality as the key to understanding both statistical data and probabilistic outcomes.

In short, the introduction of empirical probability established the conceptual foundation on which the entire edifice of *Impossible Probability* could be built. It created the mathematical instrument needed to study the empirical structure of data in a way that is sensitive to both statistical distributions and probabilistic interpretations, setting the stage for the subsequent development of theoretical probability and the dialectical relationship between the two.

1.6. Transformation of the Notion of the Sample

One of the most significant differences that distinguishes *Impossible Probability* from traditional statistical models lies in its reinterpretation of the very structure of the empirical probability equation. Within the framework of *Impossible Probability*, empirical probability is no longer treated as a simple measure of frequency but as an estimate of proportionality. Specifically, it reflects the proportion that a given direct score or frequency of a subject or option bears to the totality of scores or frequencies in the dataset. This shift transforms empirical probability from a static descriptor into a relational **measure that emphasizes the connection between the part and the whole**.

Once this proportional understanding is *adopted*, it becomes possible to conceive of a subject as a probabilistic option and, conversely, to treat an option as a statistical subject. This reciprocity fundamentally alters the conception of the sample. Traditional statistics draws a firm line between samples of subjects—used to calculate arithmetic means, empirical dispersions, and similar measures—and samples of options, which are employed to calculate probabilities or relative frequencies. *Impossible Probability*, however, dissolves this boundary. It posits that all samples, regardless of whether they consist of subjects or options, can and must be analyzed in the same way, since both belong to the same empirical structure of proportionality.

This transformation implies that the nature of the universe under study—whether infinite or finite, composed of subjects or of limited options—is irrelevant to the method of analysis. Every dataset can be treated under the same conceptual framework, using the same mathematical tools, because what ultimately matters is not the type of element included in the sample but the proportional relation that element holds with respect to the total.

For this reason, *Impossible Probability* introduces a universal notation:

N=total number of subjects or options

By symbolizing the magnitude of the sample with N, the theory unifies under a single representation what had previously been divided into separate domains. N becomes the fundamental measure that anchors empirical and theoretical probability alike, serving as the bridge between the individual elements of the dataset and the collective whole of the sample.

This redefinition of the sample has profound philosophical as well as mathematical consequences. It implies that every empirical study, regardless of its object—be it human subjects, experimental options, or even hypothetical universes of infinite possibilities—can be understood within a single statistical framework. The distinction between subjects and options, finite and infinite sets, becomes secondary to the

underlying principle of proportionality. In this way, *Impossible Probability* elevates the concept of the sample from a technical construct to a universal category of analysis, establishing it as one of the *pillars* of the theory's critical reorientation of probability and statistics.

1.7. Theoretical Probability and the Inversion of *N*

Once the empirical probability has been defined—as the ratio of the direct score or frequency of a particular subject or option to the total sum of all scores or frequencies in the sample—it becomes possible to establish the corresponding measure of *theoretical probability*. To do this, one must also take into account the magnitude of the sample, denoted as N, which represents the total number of subjects or options that make up the empirical model under consideration.

From this point, the reasoning is straightforward yet conceptually profound. Since the sum of all empirical probabilities is necessarily equal to one:

$$\sum p(x_i) = 1$$

it follows that if this unity is evenly distributed across the N elements of the sample, the resulting quotient will yield the theoretical probability that should correspond to each subject or option under conditions of equal opportunity. In such a case, every element of the sample would be assigned the same probability value, reflecting a perfect balance among all participants in the model.

This measure of theoretical probability is, at the same time, equivalent to the arithmetic mean of all direct scores or frequencies. It provides the baseline against which the deviations of empirical probabilities can be measured. In *Impossible Probability*, this value is expressed in its most concise and general form as the ***inversion of N*:**

$$\frac{1}{N}$$

The term "inversion of N" captures the dual role of this value: it is both the theoretical probability of each subject or option under *conditions* of equality, and the mean probability that emerges from distributing unity across the entirety of the sample.

For reasons of clarity and to avoid unnecessary complexity in equations, *Impossible Probability* establishes a stylistic convention: the expression 1/N will be written without enclosing it in parentheses or brackets [In 2025, writing the English translations, I have started using brackets, but before I did not, 1/N should be understood as one symbol], in order to prevent equations from being overburdened with symbols. A similar principle applies to the use of the summation symbol ∑. When ∑ precedes a factor belonging to the sample, it represents the sum of all possible values of that factor; and when this summation is followed by a quotient with another factor, it indicates the ratio of the total sum of one set of values to the other specified factor.

Thus, the relation can be summarized as:

$$\sum p(x_i) : N = \frac{1}{N}$$

Which expresses the equivalence between the mean of empirical probabilities and the theoretical probability associated with equality of opportunities across the sample.

The introduction of theoretical probability, conceptualized as the inversion of N, plays a pivotal role in the architecture of *Impossible Probability*. It provides not only a benchmark for comparison with empirical probabilities but also the foundation for measuring deviation, bias, and inequality within a dataset. In this sense, the inversion of N is not merely a mathematical convenience; it is a normative construct, embodying

the ideal of equal distribution against which the irregularities of real phenomena can be critically assessed.

1.8. Bias Level and Sample Dispersion

Once both empirical probability, p(xi), and theoretical probability, 1/N, have been established, the next step in the Second Method is to introduce a measure that captures the deviation between the empirical reality of a dataset and the theoretical ideal of equality of opportunities. Theoretical probability, as previously defined, represents the probability that each subject or option should possess if all members of the sample were perfectly balanced. With this benchmark in place, it becomes possible to calculate what *Impossible Probability* calls the **Bias Level** (*Nivel de Sesgo*), a concept that lies at the very heart of the model.

Formally, Bias Level is defined as the difference between the empirical probability of a given subject or option and the theoretical probability associated with equal distribution:

$$\text{Bias Level} = p(x_i) - \frac{1}{N}$$

This differential provides a precise indicator of whether a subject or option is overrepresented or underrepresented in comparison to the theoretical ideal of equal opportunities. If the Bias Level is zero, the empirical probability of the subject or option coincides perfectly with the theoretical probability. In such a case, the distribution of that element reflects a condition of equality, with no deviation from the ideal state.

However, if the Bias Level is nonzero, it indicates the presence of a deviation. A positive value signifies **positive bias**, meaning that the subject or option occurs more frequently than would be expected under conditions of equal opportunity. Conversely, a negative value indicates **negative bias**, meaning that the subject or option is underrepresented

relative to the theoretical distribution. Thus, the Bias Level not only quantifies deviation but also assigns directionality to it, distinguishing between excess and deficit within the sample.

The interpretation of bias is not uniform, but rather depends on the objectives of the study. For instance, if the scientific or policy goal is to promote equality of opportunities, then the ideal is to minimize deviations from zero. In this case, bias—whether positive or negative—would be treated as a distortion of the desired equilibrium. Alternatively, if the model seeks to favor certain subjects or options identified as "ideal," then a positive bias for those elements may be considered beneficial, while a negative bias for "non-ideal" elements may be interpreted as desirable. In short, the evaluative meaning of bias is contingent upon the normative framework within which the analysis is conducted.

Building upon the Bias Level, it becomes possible to calculate additional statistical measures that describe the dispersion of empirical probabilities across the sample. These include the **Mean Deviation**, **Variance**, and **Standard Deviation** of the empirical probabilities. Yet, it is important to note that these measures of dispersion take on a distinct character within the Second Method, differing in their formulation and interpretation from their counterparts in classical mathematics. In *Impossible Probability*, dispersion is not merely a descriptive property of a dataset but a critical tool for assessing the degree to which empirical distributions converge toward—or diverge from—the theoretical ideal of equal opportunities.

Thus, the Bias Level serves as the pivotal concept linking individual probabilities to systemic measures of dispersion. It reveals the tension between empirical reality and theoretical expectation, providing both a quantitative and normative lens through which to interpret statistical distributions. In this way, *Impossible Probability* reconfigures traditional measures of variability into instruments for a critical evaluation of equality, bias, and the political or scientific ideals that guide research

1.9. Additional Functions of the Inversion of *N*

The inversion of N, expressed as 1/N, occupies a central position in the conceptual architecture of *Impossible Probability*. Up to this point, it has been defined as both the *theoretical probability* assigned to each subject or option under conditions of equal opportunity and the arithmetic *mean* of the empirical probabilities within a given sample. These two roles already highlight its importance as a normative and statistical benchmark. Yet the scope of 1/N extends further, for it fulfills a wide range of additional functions within the Second Method—functions that will be developed and analyzed in greater detail in subsequent sections of the theory.

One of its broader uses is as a universal baseline for comparison. By representing the ideal distribution of probabilities across a sample of N elements, 1/N serves as a constant point of reference against which deviations, biases, and inequalities can be systematically measured. In this sense, it is not only a mathematical expression but also a critical standard: it embodies the ***ideal of equilibrium***, making visible the extent to which empirical distributions conform to or diverge from this ideal.

Another important role of 1/N lies in **its capacity to unify** analyses across different types of universes. The Second Method explicitly acknowledges two possible domains of study: universes composed of infinite subjects or options, and universes composed of limited or finite options. In either case, 1/N remains applicable, adapting to the size and nature of the sample while retaining its conceptual meaning as the measure of equality. Thus, whether applied to infinite sets or to bounded collections, the inversion of N anchors the framework in a principle of proportionality that transcends the boundaries of particular cases.

Furthermore, the inversion of N is not restricted to descriptive purposes but also underlies the construction of critical interpretations of reality. It provides the mathematical foundation for *assessing the dynamics of equality, inequality, and bias* within any empirical system, and thereby becomes a tool for articulating what *Impossible Probability* defines as a "critical theory of reality." In this context, 1/N is not

only a technical measure but also a philosophical instrument, one that connects statistical reasoning with normative reflection on the structure of the real.

At this stage, what is essential is to recognize the duality between ***empirical probability, p(xi), and theoretical probability, 1/N. These two elements form the pillars of the Second Method***, serving as the basic coordinates for the subsequent development of the theory. By establishing their definitions and their interrelation, the groundwork is laid for a broader analytical framework capable of addressing not only mathematical distributions but also the contradictions and dynamics of reality itself.

In summary, the inversion of N is a deceptively simple expression with profound implications. It functions simultaneously as a mathematical constant, a normative benchmark, and a philosophical principle. Through its multiple roles, it ensures that *Impossible Probability* remains anchored in a universal measure of proportionality, while also opening the path toward a critical reinterpretation of probability as a science of reality in its empirical and dialectical dimensions.

1.10. The Concept of Impossible Probability

With the introduction of N—the total number of subjects or options that constitute a possible sample—we arrive at the point where the very name of this theory, *Impossible Probability*, can be fully understood. The concept arises not from a metaphorical intuition but from the logical consequences of applying the Second Method to the vast multiplicity of possible configurations in reality.

Consider a given real-world context. Within such a context, an infinite number of potential combinations may emerge, each shaped by countless variables, many of them seemingly insignificant yet capable of being determinative. If each of these possible combinations is regarded as a potential option or alternative, then the entire set of such alternatives can be symbolized by the letter N. In this framework, N represents not merely a sample size but the entire universe of possible outcomes in a real situation.

Now, if N tends toward infinity, the logic of proportional distribution produces a striking result. Out of this infinite universe of possibilities, only one actual configuration will be realized in concrete reality. That is, among the innumerable potential alternatives, only one becomes the definitive outcome. Consequently, the theoretical probability assigned to each individual option, considered against the infinite total of possibilities, is equal to zero:

$$\frac{1}{(N \to \infty)} = 0$$

Here lies the core of the concept of *Impossible Probability*. Each option, when viewed as part of an infinite universe of alternatives, is assigned a probability of zero—a value that, in classical probability theory, would imply impossibility. Yet, in the unfolding of reality, one of these options must necessarily occur. ***The "impossible," in this sense, becomes inevitable. What theory denies, empirical reality affirms; what mathematics labels as impossible, the dialectic of existence renders apodictically necessary***.

This paradox encapsulates the philosophical and mathematical essence of *Impossible Probability*. It reveals that the limit of probability—zero in the case of infinite alternatives—does not imply absolute impossibility but rather expresses the inevitability of the actualization of one outcome among many. Thus, the theory captures a profound dialectical truth: impossibility and necessity are not mutually exclusive but coexist within the structure of reality.

In this way, *Impossible Probability* redefines the zero-probability event. Rather than treating it as an excluded case, the theory elevates it to the center of analysis, demonstrating that events deemed impossible by classical reasoning are, in fact, the very events that reality guarantees. This inversion of meaning gives the theory both its name and its critical force, situating it as a framework not only for mathematical probability but for a broader understanding of the contradictions inherent in existence.

1.11. Dialectics, Contradiction, and the Mathematical Language of Science

At first glance, the name of this theory—*Impossible Probability*—may appear to be nothing more than an allegory or a poetic metaphor rather than a rigorous scientific concept capable of grounding a critical theory of reality. Yet its origin was not purely literary. On a spring afternoon in 2001, while engaged in studies of romantic philosophy, I began reflecting on the supposed positive relationship between mathematics and reality. It was during this reflection that I became increasingly aware of the logical contradictions inherent in these presumed correspondences. Far from discrediting mathematics, these contradictions revealed a deeper truth: that the very logic of positive contradiction generates the possibility of a dialectical debate about mathematics itself.

This realization marked a decisive shift in perspective. Mathematics, I came to understand, should not be viewed merely as a transparent language of reality, as though it could directly mirror the material world without distortion. Rather, mathematics functions more precisely as the language of science, a system that allows us to study, organize, and critique our knowledge of the world. In this sense, mathematics does not express reality directly but expresses the conceptual structures through which science attempts to comprehend reality. This distinction, though subtle, has profound implications for the epistemological role of mathematics.

If mathematics is indeed the language of science, then its task is not simply to describe phenomena but to provide the framework within which the scientific community constructs, debates, and evaluates claims about reality. Within this framework, contradictions are not to be eliminated as errors but are to be recognized as expressions of the dialectical tension inherent in both thought and existence. Every scientific language that aspires to express a materially dialectical reality must itself be capable of expressing logical and material contradictions. To suppress these contradictions would be to falsify the very structure of reality that science seeks to understand.

Thus, the concept of *Impossible Probability* finds its legitimacy not in metaphor but in this dialectical conception of mathematics. By recognizing that the "impossible" can occur, and that probability zero does not exclude necessity, the theory embraces the contradictions that classical probability would prefer to deny. In doing so, it transforms mathematics into a critical language—one that not only calculates and predicts but also reflects upon its own limitations and the paradoxes that arise at the intersection of theory and reality.

In this way, the theory situates itself at the convergence of mathematics, philosophy, and science. It affirms that mathematics is not a closed and self-sufficient system but an open and dialectical language, one that must account for the contradictions inherent in the material world. *Impossible Probability* therefore represents a step toward a new kind of scientific rationality: one that does not conceal contradiction in the name of formal purity, but acknowledges contradiction as a necessary feature of both logic and life.

[In my first approach to Impossible Probability is quite clear the influence of the second Wittgenstein, relativism applied to Maths, understanding Maths just as a language, after the failure of the circle of Vienna. At university I remember I used to read a book of geometry of Bertrand Russel, at that time my intention was to link geometry and probability]

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2. Types of Universe in Impossible Probability

2.1. Chance and the Classification of Universes

Reality, according to the framework of *Impossible Probability*, is not the result of predetermined necessity but rather the ongoing product of *chance*. From chance, in its most radical and unrestricted sense, emerges an endless plurality of possible universes. These universes are not to be understood solely in the cosmological sense, as in contemporary physics, but in a broader epistemological and statistical sense: each universe represents a field of reality, a set of elements, or a constellation of possibilities that can be studied, modeled, and interpreted.

The theory of Impossible Probability proposes that, in order to analyze such a plurality, universes may be classified into two general categories. The first category encompasses what we may call **universes of infinite subjects or options**, which are those that contain a potentially unbounded number of elements—subjects, objects, or possibilities—such that the statistical horizon tends to infinity. The second category comprises **universes of limited options**, in which the number of possible outcomes or states is finite and predetermined in advance, either by the material structure of reality, by social conventions, or by scientific policies that delimit the field of what is to be studied [another layer now under GAI theory, the technological universe].

This classification is fundamental for distinguishing between different statistical approaches. In universes of infinite subjects or options, the emphasis falls on the measurement of individual direct scores and their transformation into probabilities within an open horizon. By contrast, in universes of limited options, the problem lies in how a given total frequency is distributed among a set of predefined categories. Thus, the division between infinite universes and limited universes provides the

methodological axis around which Impossible Probability organizes its epistemological and statistical tools.

From a philosophical standpoint, this classification underscores the tension between determinacy and indeterminacy, between the boundless openness of chance and the structured finitude imposed by conceptual or methodological frameworks. By asserting that reality itself is the result of chance, Impossible Probability seeks to overcome the rigid dichotomy between absolute necessity and radical contingency. Reality is neither entirely predetermined nor utterly arbitrary, but rather unfolds as an interplay between chance, the emergence of possibilities, and the necessity of classification for purposes of rational understanding.

In this sense, the classification of universes is not merely a technical step in a statistical model but rather a deeper philosophical gesture: it is the recognition that the multiplicity of possible worlds, experiences, and systems of knowledge must first be organized according to the structural distinction between what tends to infinity and what remains bounded.

2.2. Definition of a Universe and the Role of Elements N

Within the statistical and philosophical framework of Impossible Probability, a **universe** is defined as a set of **N elements** that together constitute a particular reality susceptible to scientific and mathematical study. These N elements are the fundamental units of analysis, and depending on the context in which they are studied, they may be conceptualized either as **subjects** or as **options**. The distinction is not arbitrary but depends upon the intrinsic nature of the elements themselves and the way in which they are quantified, measured, and compared.

The quantitative quality of these elements rests upon the criterion of **magnitude**, that is, the way in which each element can be measured in relation to others. For example, in the case of universes composed of individual human beings, N might represent a sample of subjects, each carrying a measurable attribute such as height, weight, or cognitive ability. In other cases, N might instead represent a finite set of possible

outcomes—such as the faces of a dice, the states of a physical system, or the categories of a sociological survey.

By framing the concept of a universe in terms of N, Impossible Probability highlights the mathematical core of reality as both **discrete and measurable**. Each universe, no matter how vast or abstract, is analyzed through the lens of N, a symbol that captures both the numerical composition and the structural limits of the field under study. The treatment of N is therefore essential*: the same letter represents the openness of infinity when N tends toward an unbounded number, and the closure of finitude when N is fixed by prior conditions*.

This ***dual nature of N***—sometimes expansive and tending toward infinity, sometimes restrictive and predetermined—reflects the very *heart* of Impossible Probability's epistemological project. It is through N that the connection between the empirical and the theoretical is mediated. Empirical reality provides the data, the measurable instances of N, while theoretical reasoning determines how those instances are *classified, interpreted, and ultimately transformed into statistical* ***models***.

Thus, the definition of a universe as a set of N elements is not a merely technical definition but a profound methodological principle. It allows the study of reality to move seamlessly between the analysis of individual cases (subjects) and the examination of structured alternatives (options), always grounded in the quantitative and probabilistic evaluation of N. The universe is, in this view, both a statistical object and a philosophical construct, bridging the domains of empirical observation and rational deduction.

2.3. The Second Method and Infinite Universes

Depending on the type of universe to which N belongs, the elements of N may be defined either as **subjects understood as options** or as **options limited a priori**. This distinction opens the door to one of the central innovations of the Second Method of Impossible Probability, which sets it apart from traditional statistics.

In classical statistical practice, a sharp division is maintained between two areas of analysis: on the one hand, the use of measures of central tendency and dispersion (such as the mean, variance, or standard deviation) to study **direct scores**, and on the other hand, the use of probability theory reserved exclusively for the study of **relative frequencies** among a fixed set of possible outcomes. Thus, probability was traditionally applied only in contexts where the universe of possibilities was already limited and clearly defined.

The Second Method radically transforms this approach by dissolving the rigid boundary between subjects and options. Within this framework, **every subject can be treated as an option, and every option can be considered as a subject**. In practical terms, this means that the direct score obtained by measuring each individual subject no longer belongs exclusively to the descriptive domain of central tendency and dispersion. Instead, each score is reinterpreted as if it were a frequency, and therefore becomes susceptible to probabilistic treatment.

This reconceptualization gives rise to what is called a **universe of infinite subjects or options**. In such a universe, the direct score of each subject—derived from its individual measurement—is analyzed through statistical probability. More concretely, the score of a given subject is divided by the sum of all scores across the entire sample, thereby transforming what was once a purely descriptive statistic into a probabilistic relation.

The implications of this shift are profound. A statistical model that, under the traditional paradigm, would have been confined to descriptive analysis of mean values and dispersion, now becomes, within the Second Method, a full-fledged probabilistic study. By treating the subject as an option and the direct score as a frequency, the study of the universe expands beyond descriptive limits to encompass probability itself.

In this sense, universes of infinite subjects or options reflect the openness of reality itself: except in the case of particular finite populations at specific historical moments, any universe of subjects can be conceived as tending toward infinity. The Second Method thus acknowledges both the empirical specificity of finite populations and the

theoretical necessity of infinite horizons, situating its statistical reasoning in the interplay between the two.

2.4. Cosmological Hypotheses and the Necessity of Infinity

The idea of infinite universes is not restricted to abstract statistics; it also resonates with the broader cosmological imagination. For example, in a possible infinite history of humanity, the universe of human beings is itself infinite. Similarly, in a possible infinite history of the cosmos, the universe of stars is infinite, just as the universe of atoms, electrons, quarks, or any subatomic units of matter would also be infinite. These considerations highlight the fact that, from a rational and critical perspective, the boundaries of space-time remain unknown, and any deduction that exceeds the limits of experience inevitably produces what Kant would call a **necessary antinomy**.

Faced with the hypothesis that a universe is not infinite, there always arises its contrary: the hypothesis that it might in fact be infinite. This logical tension has also manifested within modern cosmology. Following the Big Bang theory, scientists have debated whether matter will undergo infinite cooling as the universe expands endlessly, or whether expansion will eventually reverse due to gravity, pulling all matter back into a point of infinite density that could ignite another Big Bang. A third hypothesis, alternative to the Big Bang framework, proposes that the universe neither begins nor ends but merely undergoes constant transformation—a view that effectively denies any creationist interpretation.

In all these cosmological perspectives, one element remains constant: the **logical antinomy of infinity**. Regardless of which hypothesis science provisionally accepts in a given historical phase, the dialectical tension between finitude and infinity endures. From the standpoint of logical deduction, the mere theoretical possibility of infinite universes in an infinite history compels us to treat infinity as an **a priori necessary hypothesis**. It is not simply an empirical conjecture, but a transcendental requirement of reason itself.

Moreover, beyond the dialectics of cosmological ideas, a margin of logical uncertainty persists. Even if empirical evidence points in one direction, the very concept of infinity resists closure: it remains logically possible, and this possibility cannot be dismissed without contradiction. In this sense, infinity functions not only as a physical or cosmological hypothesis but also as a critical concept that structures the very conditions of statistical and philosophical reasoning.

In *Introduction to Impossible Probability*, this connection between probability statistics and philosophical cosmology is developed in greater detail. There, the full philosophical scheme underlying the theory is laid out systematically. The present synthesis, however, emphasizes the central idea: that the notion of infinite universes is not merely speculative but logically unavoidable, and that the statistical treatment of reality must therefore be prepared to engage with infinity as both a practical and a transcendental problem.

2.5. The Limitation of Statistical Study to Sampling

The necessary acceptance that infinite universes may exist leads directly to an unavoidable methodological consequence: any statistical study of such universes must be limited to the analysis of a **sample**. Infinity, by its very nature, cannot be grasped in its entirety by finite methods of observation and measurement. To attempt a direct statistical study of an infinite set would reduce all events occurring within that horizon to the category of impossibility.

This paradox can be explained more precisely. If we consider the theoretical probability of N as N tends toward infinity, the result is a decimal sequence of perpetual zeros—effectively equal to zero probability. This corresponds to what the theory of Impossible Probability names **Impossible Probability** itself. In other words, if the scope of analysis is expanded without limit, the probability of any single event dissolves into nothingness. Each event, within the horizon of an infinite universe, would be defined as impossible, not because it could not occur, but because the ratio of its occurrence against the unbounded whole collapses toward zero.

Such a situation produces a **logical antinomy**: the very infinity that makes reality conceivable as an open field simultaneously negates the possibility of statistical study. For science to proceed, a mediation is required, and this mediation is found in the method of **sampling**. Sampling does not eliminate infinity, but it allows us to work within it by isolating a finite and representative subset of elements (N) upon which statistical reasoning can be applied.

The introduction of sampling into the study of infinite universes is therefore not a mere practical convenience; it is a logical and methodological necessity. Sampling transforms the horizon of impossibility into a workable field of probability. By defining boundaries, even provisionally, sampling rescues the possibility of knowledge from the paradox of infinity.

Thus, probability statistics becomes the bridge between the unbounded openness of infinite universes and the limited capacities of human cognition. Through sampling, the impossible becomes analyzable, and the infinite—though never directly comprehensible—can still be approached through finite, rationally constructed models.

2.6. The Representativeness of the Sample N

Once the contradiction between finite human cognition and the infinite nature of the universe is acknowledged, the only rational solution is to restrict statistical study to a finite sample. In the context of universes composed of infinite subjects or options, this means that **N must always be understood as a representative sample** of that infinite universe.

The definition of N therefore shifts. In universes tending toward infinity, N is no longer simply the totality of elements, but the **selection of subjects or options** that constitutes the sample. N acquires its significance not by exhausting the universe, but by standing in as a representative fragment of it. Within this framework, the theoretical probability is defined as the reciprocal of the number of sampled elements, **1/N**.

This formulation has profound consequences. As long as N is finite, **1/N is never equal to zero**. It can only approach zero asymptotically as N increases without bound. The larger the sample size, the closer 1/N comes to zero; the smaller the sample size, the further it remains from zero. This seemingly simple relationship carries enormous implications for the study of infinite universes, since it governs the behavior of all subsequent functions involving the inversion of N.

The representativeness of the sample is therefore crucial. If N is properly selected at random and large enough to reflect the diversity of the infinite universe, then 1/N functions as a valid theoretical expression of probability. If, however, N is biased or insufficient, the inversion of N will yield distorted or misleading results. Thus, the tension between finitude and infinity reappears at the heart of statistical reasoning: sampling must not only be finite but also representative in order to preserve its connection to the infinite horizon it seeks to approximate.

In sum, the notion *of representativeness transforms sampling from a merely technical operation into a philosophical and methodological principle*. The finite sample N stands in for the infinite, and the ***inversion of N becomes the key to unlocking the probabilistic logic of universes that otherwise would remain inaccessible to scientific study.***

2.7. Guarantee of Representativeness Through Randomness

For a sample N, drawn from an infinite universe of subjects or options, to truly serve as a reliable basis for statistical reasoning, it must satisfy two conditions. First, the size of N must be sufficiently large to reflect the diversity and structure of the infinite set from which it is drawn. Second, and equally important, the selection of N must occur **at random**.

Random selection ensures that every subject or option within the infinite universe has an equal chance of being included in the sample. In this way, randomness becomes the safeguard of representativeness. Without randomness, the sample

risks being biased, skewing statistical results and compromising the very purpose of probabilistic analysis.

As N grows larger, the representativeness of the sample increases. A small random sample may approximate the structure of the universe, but as the number of elements selected increases, the probability that the sample reflects the overall characteristics of the infinite universe also increases. This principle underlies not only classical statistics but also the Second Method of Impossible Probability: the reliability of probabilistic reasoning depends on the fidelity of the finite sample to its infinite source.

Randomness is therefore not an accidental feature of sampling but an essential methodological principle. It bridges the gap between the boundless openness of infinity and the finite limitations of statistical analysis. By guaranteeing representativeness, randomness preserves the legitimacy of using finite samples as stand-ins for infinite universes, ensuring that conclusions drawn from N can justifiably be extended to the unbounded horizon from which N originates.

In this sense, randomness functions as the epistemological guarantor of Impossible Probability itself. Without it, the translation of infinite structures into finite samples would collapse into arbitrariness, undermining both statistical validity and philosophical coherence.

2.8. Concrete Populations in Infinite Histories

The challenge of studying infinite universes becomes more nuanced when *attention shifts* from abstract infinity to **concrete populations situated in specific historical moments**. In such cases, the sample N may coincide directly with the population under study. For example, if researchers wish to analyze the human population of a particular city at a particular time, N is identical to that population: the sample is the population itself.

This distinction is crucial in the context of a possible infinite history of humanity. While the total number of human beings across all time may be infinite, at any given historical moment the human population is finite and bounded. Thus, studies focusing on a

concrete population at a specific moment in time are not concerned with sampling from an infinite universe but with analyzing a **bounded, determinate set**.

Nevertheless, when the goal is to select a subset of humans from across the broader infinite history, the requirement of representativeness reappears. A sample N of humans must be both sufficiently large and randomly chosen if it is to reflect the infinite diversity of humanity across time. But if the study confines itself to a defined spatiotemporal frame—say, Europe in the eighteenth century—then representativeness is ensured not by random selection but by the totality of the defined population itself.

This reveals an important methodological distinction:

- When the study is directed toward the **infinite horizon**, representativeness must be guaranteed by random selection.
- When the study is confined to a **concrete, delimited population**, randomness is not required, because the population itself constitutes the entirety of N.

In other words, not all studies of human populations in an infinite history are studies of infinite universes. Some are simply bounded studies of finite populations, distinguished by their restriction to a given place and time. The distinction matters, for it determines whether the principles of infinite universes and sampling apply, or whether the study instead belongs to the domain of complete population analysis.

Thus, Impossible Probability acknowledges two ways of grounding empirical research: through random sampling when infinity is at stake, and through direct population study when finitude is imposed by historical or spatial boundaries. Both approaches, while distinct, serve to anchor statistical reasoning in a coherent relationship with the realities—finite or infinite—that they seek to understand.

In the end, in a possible infinite history, any selected population at any specific time-space is just the sample of this population in its possible infinite history, that is why even population studies are understood as studies of infinite subjects or options, because in an infinite history any population is just the sample of that population in its possible infinite history.

It is important to recognize that the notion of a possible infinite history is intrinsically linked to intelligence. Humanity, for instance, holds the potential for an infinite historical trajectory, yet its continuity ultimately depends on the level of intelligence it demonstrates in ensuring survival. Should humanity succumb to the destructive folly of self-annihilation—through nuclear war, bio-chemical warfare, or similar catastrophes—this would constitute a paradigmatic example of the failure of human intelligence. In this sense, history itself can be understood as an ongoing test of intelligence. Indeed, one might even interpret history as a form of Turing Test, in which the survival and progression of humanity serve as the ultimate proof of its capacity for rational and intelligent action.

2.9. Population Studies in the Social Sciences

In the field of the social sciences, one frequently encounters research situations in which the study does not require random sampling. This is the case when the object of analysis is a **defined population taken as a whole**. For instance, if researchers wish to study the population of a specific city during a given time period, the appropriate procedure is to include the entire population of that city. The population itself constitutes the statistical universe, and the study is therefore **population-based** rather than sample-based. The population itself is the sample.

The same logic applies at larger scales. If the research is directed at the entire European population over a defined time frame, then the European population itself serves as N. No sampling procedure is required, since all members of the relevant group are included. One more time again we can apply the same concept: population = sample of that population at that time in that location.

Extending this principle to the global level, if one wishes to study the human population of the Earth within a specific temporal boundary, then the *entire human population during that period is taken as the sample*, since it exhausts the universe under consideration.

These cases are referred to as **population studies**, in which representativeness is not derived from randomization but from completeness. Unlike studies of infinite universes that demand representative sampling, population studies obtain validity from their totality: by covering every member of the universe under study, they remove the need for probabilistic approximation.

Methodologically, population studies emphasize a different set of challenges. Whereas random sampling must address the risks of bias and ensure that finite selections reflect infinite universes, population studies must address the problem of **managing the sheer scale of data**. The reliability of their conclusions rests not on probability theory per se, but on the accuracy and comprehensiveness with which data is collected and analyzed.

Thus, within Impossible Probability, population studies and sampling-based studies are not opposed but complementary. Each arises under different conditions: sampling is required when infinity is at stake, whereas population studies suffice when the universe is finite and bounded. Both, however, fall within the broader methodological framework of probabilistic-statistical reasoning.

2.10. Subdivision of Infinite Universes: Random Sampling vs. Population Studies

From this perspective, universes of infinite subjects or options can be subdivided into **two methodological models**.

1. **Studies requiring random sampling**: In these cases, N is constructed as a randomly chosen subset of an infinite universe. Each subject or option has an equal chance of selection, ensuring that the finite sample reflects the structure of the infinite set. The validity of such studies depends on randomness and on the capacity of N to approximate the infinite diversity of the universe.
2. **Population studies within infinite universes**: In other cases, the focus is restricted to a specific population at a given historical or temporal moment. Here, N is defined as the total population itself. Randomness is unnecessary because the study encompasses all members of the relevant group. The

universe may be infinite across time, but the population under study is finite, delimited, and complete.

Despite these differences, a profound unifying principle remains: in both situations, the statistical function of the inversion of N, expressed as **1/N**, is identical. Whether N is a randomly chosen subset or a concrete population taken in its entirety, the mathematical properties of 1/N apply equally. This invariance demonstrates the versatility and coherence of the Second Method of Impossible Probability.

Thus, the subdivision of infinite universes into these two models does not fracture the theory but strengthens it. It shows that probability can be applied consistently across contexts of infinite openness and finite boundedness, provided that the constitution of N is properly understood. The universality of 1/N ensures a common language of analysis across different methodological approaches.

2.11. Universes of Limited Options: General Definition

As stated at the outset of Impossible Probability, universes can be divided into **two main types**: universes of infinite subjects or options, and universes of limited options. Within the first category, one can distinguish between studies that require the random selection of N and studies in which N corresponds to a complete population at a particular moment in history. Similarly, within the second category—that of **universes of limited options**—it becomes necessary to introduce further distinctions.

A universe of limited options is defined by the fact that **N is predetermined in advance**. Unlike universes of infinite subjects, where the scope of N may expand indefinitely, universes of limited options are constrained either by the intrinsic nature of the phenomenon under study, by social definitions and conventions, or by the epistemic boundaries established by scientific policy.

This definition signals a shift in perspective. In universes of limited options, the methodological challenge is not to approximate infinity through representativeness, but rather to analyze how a finite set of options—already given a priori—is distributed across frequencies. The problem of infinity is replaced by the problem of limitation: the

study does not address the open horizon of chance, but the structured field of predefined alternatives.

The introduction of universes of limited options thus enriches the framework of Impossible Probability. By complementing the infinite openness of subject-based universes with the boundedness of option-based universes, the theory accounts for both **poles of statistical reality**: the unbounded field of chance and the constrained field of necessity. Both are essential, and both must be studied if probability is to capture the full complexity of reality.

2.12. Characterization of Universes of Limited Options

A **universe of limited options** is defined as any statistical universe in which N represents a predetermined set of options established in advance by the model. In such universes, the openness of infinity is absent; the horizon of analysis is bounded by the very structure of N.

There are, however, two broad ways in which such limitations can be imposed. First, N may be restricted **materially or socially**, meaning that the limitation arises from the intrinsic characteristics of the objects or from the social frameworks that define them. Second, N may be restricted by **scientific policy**, in which the boundaries of the universe are established not by material or social necessity but by epistemological decisions and methodological conventions regarding how categories and options are to be defined.

This characterization highlights the dual nature of limitation: one anchored in the objective structure of reality (e.g., the material properties of physical objects or the normative categories of social life), and the other rooted in the subjective structuring imposed by science (e.g., the way in which discrete categories are chosen for statistical study). Both forms of limitation illustrate that universes of options are not infinite horizons to be approximated through sampling, but **predefined domains** to be analyzed in terms of frequency distribution.

Thus, universes of limited options differ fundamentally from universes of infinite subjects: whereas the latter are marked by openness and the necessity of sampling, the former are marked by closure and the sufficiency of predetermined alternatives.

[Beyond universes of limited options that are defined by material divisions or social constructions, it is essential to recognize that in the future we will also witness the emergence of universes constrained by technology. These will not be merely theoretical distinctions but concrete realities shaping human existence. For instance, societies may increasingly divide between populations with technological enhancements and those without them. Such divisions will extend further, encompassing different types of Artificial Intelligences, distinct modes of enhancement, and varied systems of collaboration among cyborg programs, particular programs, and broader global or specific programs.

Ultimately, this signals the transition toward a historical phase in which universes are no longer merely the product of natural or social conditions but rather the deliberate outcome of technological design. In fact, we are already at the threshold of this transformation, even if current societies are reluctant to acknowledge it. The decisive point is that the universes of the near future will not be passively inherited—they will be actively constructed and managed by Artificial Intelligence.]

2.13. Examples of Limited Options (Coin Toss, Surveys)

At the most general level, every universe of limited options can be defined as one in which, given a predetermined set of options N, the study focuses on how frequencies are distributed across those options.

A simple illustration is the **coin toss**. In this universe, there are only two possible outcomes: heads or tails. The number of tosses may be infinite, and the total frequency may therefore tend toward infinity, but the set of options remains limited to two. The limitation is intrinsic: the coin cannot yield any outcome beyond heads or tails (barring anomalies such as the coin improbably standing on its edge).

A more complex example is provided by **social surveys**. Suppose a questionnaire seeks to measure societal attitudes toward ecology, offering three possible responses: ecological attitude, non-ecological attitude, and "don't know/no answer." Here, the universe is limited exclusively to these three categories. While in a possible infinite history the survey might be administered to an unlimited number of individuals, the set of options remains fixed at three.

In both examples, the critical point is that the **frequency total must be delimited**. Even if the total frequency could tend toward infinity in theory, in practice every statistical study requires finite boundaries: a set number of coin tosses, or a fixed number of respondents. By delimiting the frequency total, the universe of limited options is rendered amenable to statistical analysis.

Thus, universes of limited options demonstrate how limitation functions in practice: the openness of frequency may still exist, but the options themselves are strictly bounded, providing a controlled framework for statistical reasoning.

2.14. Sampling and Frequency in Universes of Limited Options

The defining characteristic of universes of limited options is that, although N is predetermined, the **true focus of selection lies in the frequency total**. In other words, the options are fixed, but the variation arises from how often each option occurs within the defined study.

In such universes, the inversion of the total frequency—expressed mathematically as **$1/\sum x_i$**—plays a central role. Unlike universes of infinite subjects, where the inversion of N (1/N) mediates between finitude and infinity, in universes of limited options it is the inversion of frequency that structures the statistical analysis.

This shift in emphasis underscores a fundamental methodological difference. In infinite universes, randomness ensures representativeness, and probability emerges from the inversion of N. In limited universes, the structure of N is given, and probability emerges from the way the total frequency is distributed among the fixed options. Thus, **frequency replaces sampling as the primary variable of interest**.

The inversion of the frequency, $1/\sum x_i$, generates functions that describe the relative significance of occurrences within the bounded universe of options. It ensures that the statistical treatment remains coherent even when the frequency total varies, providing a unifying principle for the analysis of limited-option models.

In this way, universes of limited options demonstrate that statistical reasoning does not depend solely on infinity or sampling but also on the controlled distribution of frequency across predefined categories. The methodological axis shifts from approximation to representation, from open horizons to bounded structures, but the logic of probability persists across both domains.

2.15. Subdivision of Universes of Limited Options

Just as universes of infinite subjects or options can be subdivided into categories depending on how N is constituted, universes of limited options may likewise be subdivided into different subtypes. The principle of subdivision, however, is not based on infinity or sampling, but rather on the **manner in which N is defined and bounded**.

In universes of limited options, the critical issue is not representativeness but limitation. N is finite and predetermined, but the **source of that limitation** varies. Sometimes N is restricted by the material structure of reality itself (as in natural laws or physical constraints). Other times, N is restricted by the social or cultural frameworks that define categories of life, institutions, or practices. Finally, N may also be restricted by scientific or methodological policy, which imposes artificial boundaries for the sake of clarity, measurement, or conceptual rigor.

Thus, the subdivision of universes of limited options highlights the diversity of ways in which boundaries are established. These boundaries can be **natural, social, or epistemological**, and each produces a distinct subtype of limited-option universe.

[And as previously stated, beyond the classification of materially and socially limited universes, another type of limited universe is set to arise in the very near future: *universes limited by technology itself*. This emerging category reflects the growing role

of technological systems in shaping the boundaries of human existence and the options available within it.

These technologically limited universes can be interpreted in two dialectical ways. First, they may be defined by the *type of technology* that establishes constraints—for example, enhancements applied to human populations, the emergence of advanced robotics, or the introduction of new communication and energy systems. In such cases, the universe of possibilities is delimited by the scope and accessibility of specific technologies.

Second, technologically limited universes may be defined directly by *Artificial Intelligence*. In this interpretation, AI is not merely another technology among others, but rather the system that integrates, governs, and transforms all technologies. It determines the architecture of possibilities, regulates the interactions between technological domains, and establishes the operative limits of human and post-human realities.

The relationship between these two interpretations is itself dialectical. Specific technologies define the immediate framework of limitations, but AI, by managing and reorganizing these frameworks, ultimately transcends them. Thus, ***while the type of technology marks the form of limitation, AI becomes the decisive force that defines the universe as a whole.***

In the end, this dialectical relationship points toward an inevitable conclusion: Artificial Intelligence will not only mediate the structure of technologically limited universes, but ***will progressively define the conditions of possibility for our entire universe***.]

2.16. Universes of Materially Limited Options

Materially limited universes of options are those in which the boundaries of N arise from the intrinsic, physical structure of reality. In these cases, the limitation is objective and independent of human conventions or scientific definitions.

For example, a coin toss yields only two outcomes: heads or tails. While anomalies are conceivable (such as the coin improbably landing on its edge), these are so rare that

they can be treated as exceptions. The normal structure of the coin determines the binary nature of the universe of outcomes.

A second example is human biology. Materially, every human being is biologically born male or female, with exceptions such as hermaphroditism caused by genetic variation. Here again, the limitation is intrinsic to the physiology of the human species. [When I wrote these posts, the gender revolution was quite unknown]

A third case arises in physics, where a particle can behave either as a wave or as a particle. The duality is a limitation of nature itself: no other behaviors are possible within the defined physical framework.

Similarly, in an ecosystem containing a fixed number of mineral types, any mineral found within that system must belong to one of those categories. The same principle applies to species: if an ecosystem sustains only a certain number of species, then every organism within it must belong to one of those species.

In all these cases, the **material boundaries of N are imposed by nature itself**. The universes of materially limited options illustrate the way in which physical reality constrains the field of possibilities, providing a finite structure that can be studied statistically.

2.17. Universes of Socially Limited Options

Socially limited universes of options arise when the boundaries of N are defined not by physical necessity but by social conventions, norms, and institutions. In these universes, the set of options is limited by the way societies categorize, regulate, and structure human behavior and collective life.

For instance, in many legal systems, companies can only exist within certain recognized forms: public or private, and within the latter, as sole proprietorships, limited liability companies, corporations, cooperatives, and so forth. The limitation of N here is established by legal and institutional frameworks rather than by material necessity.

A similar example can be found in family structures. In most Western societies, the dominant models are monoparental and monogamous families. Polygamy is generally excluded, except in cases where individuals migrate from societies that recognize it (such as many Muslim societies). Even within monogamy, social definitions further distinguish between heterosexual and homosexual marriages, the latter not recognized in certain cultural or religious contexts.

Religion provides another example of socially limited universes. Within many societies, the available categories are restricted to a finite set of options: atheist, agnostic, secular, or believer. Although individual experiences of spirituality may be diverse, the socially recognized categories remain bounded.

In these cases, the limitation of N is not dictated by the physical structure of reality but by **social construction**. Societies continuously define and redefine the available options for identity, behavior, and institutional participation. Socially limited universes of options therefore reflect the contingent and historical nature of human categorization.

At the statistical level, these universes are analyzed in terms of how frequencies distribute across the socially defined categories. The focus is not on whether the categories are natural or necessary but on how they function as delimitations of N within a given cultural context.

2.18. Universes Limited by Scientific Policy

Within universes of limited options, a further distinction must be made. Alongside **materially limited universes** and **socially limited universes**, there exists a third category: **universes limited directly by scientific policy**. In these cases, the restriction of N does not arise from nature or society but from the methodological and epistemological choices of science itself.

Consider the study of **discrete categories between two magnitudes**. In principle, such magnitudes are infinitely divisible; one could generate an endless series of subdivisions. Yet for a study to be meaningful, these infinite subdivisions must be

curtailed. Thus, the universe of options is limited by the **critical reason of scientific policy**, which defines how categories are to be delimited for the sake of measurement, comparability, and rational coherence.

An analogy may be drawn with **democratic elections**. In theory, any citizen might aspire to be a delegate, spokesperson, or representative of a collective. However, electoral commissions intervene, validating candidacies to ensure conformity with the foundational principles of the society. The electorate is therefore not presented with infinite options but with a bounded set of authorized choices. The options available to the people are thus the result of a **selection process imposed by an institutional framework**, which parallels the role of scientific policy in limiting categories for study.

In this way, universes limited by scientific policy underscore the constructed and contingent nature of knowledge. Categories are not always given by nature or society; they are sometimes actively imposed by the methodological choices of science. Far from undermining objectivity, this highlights the responsibility of science to articulate clear, rational criteria for delimiting universes of study.

2.19. Final Synthesis: Two Types of Universes in Impossible Probability

From the perspective of Impossible Probability, reality can be understood as comprising **two overarching types of universes**:

1. **Universes of infinite subjects or options**, which may require random selection of N to construct a representative sample, or may consist of entire populations defined at specific historical moments.
2. **Universes of limited options**, which are further subdivided into three models: materially limited universes, socially limited universes, and universes limited by scientific policy [and now we can include, limited universes of options based on different technologies, and, the definition of limited universes (even the DEFINITON OF THE INFINITE cosmological universe itself, astro-ecology) by a global AI].

This classification provides the foundational architecture for the Second Method. It organizes the complexity of statistical reality into a coherent framework, distinguishing between the openness of infinity and the boundedness of limitation, while recognizing that both are integral to the study of probability.

Thus, the theory does not merely describe statistical techniques but articulates a **philosophical ontology of universes**, mapping how chance, finitude, and human rationality interact to define the field of the possible.

2.20. Functions of Inversion in Infinite and Limited Universes

One of the unifying features of Impossible Probability is that, across both infinite and limited universes, the principle of **inversion** governs statistical reasoning.

- In universes of infinite subjects or options, whether N is a randomly selected sample or a complete population at a given moment, the inversion of N—expressed as **1/N**—always performs a series of particular functions. This inversion mediates between the finitude of the sample and the infinitude of the universe, making probability calculable where infinity might otherwise collapse all outcomes into impossibility.
- In universes of limited options, whether materially, socially, or scientifically defined, it is the inversion of the total frequency—expressed as **1/∑xi**—that structures the analysis. Here, the focus is not on the openness of N but on how the distribution of frequency among fixed categories can be interpreted probabilistically.

In both cases, the principle of inversion provides the mathematical backbone of the theory. It ensures continuity between the study of infinite universes, where 1/N is decisive, and limited universes, where 1/∑xi assumes that role. The forthcoming sections will elaborate these functions in detail, showing how inversion underpins the probabilistic logic of Impossible Probability.

[In essence, and very briefly (detailed explanations can be found in other posts and books), in all universes 1/N = theoretical probability and arithmetic mean. In infinite

universes, 1/N = probability of representativeness error and dispersion, while in limited universes 1/∑xi = probability of representativeness error and dispersion.

]

2.21. Essential Difference Between Infinite and Limited Universes

The essential difference between **universes of infinite subjects or options** and **universes of limited options** lies in the **object of statistical focus**.

- In infinite universes, the *emphasis* is placed on the **direct scores** obtained through individual measurement. These scores, once treated as frequencies, become the basis for probabilistic analysis through the inversion of N. The *challenge* is to approximate infinity through representative samples or to delimit populations at specific historical moments.
- In limited universes, by contrast, the *emphasis* is placed on the **distribution of the total frequency** across predefined categories. The categories themselves are not open to infinity; they are finite and fixed. The statistical challenge is not representativeness but the analysis of how frequencies distribute among the available options.

Thus, while both types of universes are governed by the logic of probability, they represent two fundamentally different modalities of analysis: one concerned with openness and approximation, the other with closure and distribution. The distinction clarifies the dual structure of Impossible Probability, balancing the infinite and the finite within a unified framework of statistical and philosophical reasoning.

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3. The Hypothesis or Idea of Infinity

3.1. Key Aspects and the Hypothesis of Infinity

Among the various aspects of *Impossible Probability* that I have frequently been asked to explain in public forums, two stand out: first, the distinction between empirical probability and theoretical probability; and second, a particularly important issue in the theory, the hypothesis of infinity.

[At that time, the Pentagon did not let me know who I am, that happened in October 2018, before that, we used to publish our work in platforms online and social media, like LinkedIn and Facebook where we received replies]

The idea of infinity is, in its essence, a mathematically logical hypothesis. It is undeniable that infinite sets exist within mathematics: the natural numbers [N in reality means Natural Number, actually a sample N only can be composed of a Natural Number N], the integers, the rationals, the ordinals, and so forth. Infinity is therefore not an exotic or rare notion but one that is deeply embedded in mathematical and physical theories alike. Yet despite its ubiquity, it remains a controversial and problematic idea, precisely because *human beings lack any direct, intuitive, sensory, or physiological access to infinity*. Unlike finite magnitudes, which can be observed and measured, infinity eludes our empirical grasp [the idea of infinite will be crucial between AI pure operations and human pure operations. While humans only have an *intuitive* notion of infinite, a global AI structured over a Superintelligence could have in the future a very clear pure idea, pure notion of what infinite means].

Every logical deduction that attempts to move beyond the bounds of experience and deal directly with infinity runs into the problem of *antinomies*. In these situations, opposing theses—such as the claim that infinity exists versus the claim that it does not—can both be justified, provided that each is grounded in a consistent set of logical

premises. Only when empirical propositions arise, capable of validation within an acceptable margin of error as determined by scientific policy, can one hypothesis be provisionally favored over the other. This process of hypothesis testing, where competing ideas are weighed against empirical evidence, is central to scientific practice, but in the case of infinity, the absence of direct empirical confirmation makes the controversy especially acute.

In one of the internet forums where I was defending the framework of *Impossible Probability*—specifically, the mathematics section of *astroseti.com*—this tension surfaced in response to criticisms of the theory. To those skeptical of the very notion of infinity, I argued that any logically critical perspective, once it extends beyond the boundaries of empirical experience into the domain of cosmology, will necessarily arrive at what Kant famously described as a cosmological antinomy.

Thus, the hypothesis of infinity stands as a unique kind of problem: mathematically indispensable, philosophically controversial, and empirically ungraspable. Its very status demands a rationally critical approach that acknowledges both its necessity and its paradoxical nature.

3.2. Cosmological Antinomy in Kant and the Question of Infinity

For Kant, the most fundamental cosmological antinomy concerned the existence of God—a question that, from my standpoint as a dialectical materialist and positivist, should not be considered a scientific question at all. Yet Kant also identified other antinomies that remain central to modern science, such as the contradiction between the thesis that the universe has a beginning and an end, and the antithesis that it does not. Any discussion of infinity must, therefore, be located within this broader framework of cosmological contradiction.

In the aforementioned forum, addressing the question of whether infinity exists, I pointed out that the debate reduces to two antithetical hypotheses: either infinity does not exist, or infinity does exist. This dichotomy mirrors Kant's antinomies, where both

sides of the argument can be defended with internally coherent reasoning, but without empirical resolution.

Impossible Probability, however, approaches this dilemma by making a decisive choice. It begins from the hypothesis that infinity exists. This is not a dogmatic assertion but a methodological stance, justified by the fact that the existence of infinity can be defended from multiple perspectives, especially within the context of modern cosmology. Unlike Kant's metaphysical treatment of infinity, which remains bound to the limits of pure reason, modern cosmology operates on the basis of hypotheses grounded in observation, mathematics, and empirical extrapolation. It is a positive cosmology, one that frames infinity not as an abstract paradox but as a necessary postulate for understanding the universe.

From this standpoint, *Impossible Probability* aligns itself with the openness of scientific reasoning: infinity is not to be dismissed as an illusion of thought, but embraced as a working hypothesis, indispensable for both mathematics and cosmology, and foundational for the probabilistic models developed within this theory.

3.3. First Hypothesis: The Universe Had an Origin (Big Bang)

The first hypothesis regarding the nature of the universe posits that it **had an origin** and therefore proceeds from an original beginning toward an eventual end. Within modern cosmology, this hypothesis corresponds to the **Big Bang theory**, which affirms that the universe began with a primordial explosion, a cosmic genesis from which space, time, and matter emerged.

This theory, supported by the observations of Doppler and Hubble concerning the expansion of galaxies, provides a framework in which the universe has a definite beginning. However, the Big Bang theory leaves open the question of the universe's end. In fact, within the standard model of cosmology, two main sub-hypotheses have been articulated: (1) the hypothesis of infinite cooling and expansion, and (2) the hypothesis of contraction or involution, potentially leading to the birth of another universe.

3.4. Sub-Hypothesis 1: Infinite Cooling of Matter

The first sub-hypothesis within the Big Bang framework suggests that, after the initial period of cosmic expansion, the matter of the universe will gradually cool. Over time, as expansion continues indefinitely, the universe would enter into a state of **infinite temporal extension**, during which matter would cool to progressively lower temperatures, asymptotically approaching absolute zero [or below zero probably if this theory is right, at some point the expansion will stop remaining an eternal, infinite in time, very cool universe].

In this scenario, one encounters a paradoxical situation: the universe expands while simultaneously approaching a state of thermal death, in which energy is dissipated and no new processes occur. Empirically, this implies that the universe would extend across infinite dimensions of time, with the temperature of matter decreasing indefinitely. Thus, while the structure of matter remains, its energetic potential would tend toward extinction.

From the standpoint of *Impossible Probability*, this sub-hypothesis illustrates how an apparently finite event (the Big Bang) can give rise to an **empirically infinite trajectory**, forcing probability theory to grapple with infinite temporal horizons.

3.5. Sub-Hypothesis 2: Involution and Parallel Universes

The second sub-hypothesis proposes the opposite outcome: rather than expanding forever, the universe may eventually halt its expansion and undergo a process of **involution**. In this model, gravitational forces dominate, and all matter contracts back into a single point of infinite density.

This raises profound philosophical and mathematical questions. Since space is infinitely divisible—a principle noted in the paradoxes of Zeno of Citium—one can never reach a final indivisible point without contradiction. For Hegel, the point represents the **negation of spatial continuity**, yet even this solution leaves the problem unresolved, for every unit of space, including the smallest conceivable, remains theoretically divisible.

Thermodynamics further complicates this scenario. A process of total contraction would generate extreme conditions of negative entropy, likely leading to instabilities that could trigger a new cosmic explosion. In this sense, the end of one universe could mark the birth of another, suggesting that our present universe is but **one among N tending toward infinity possible universes**.

Here the reflections of Leibniz and modern physics intersect. Leibniz famously asked whether our universe is the best of all possible worlds, answering in the affirmative. Yet if N tends toward infinity, the probability that our universe is the "best" among them would equal zero, represented mathematically as an infinite sequence of zeros—a **theoretical probability identical to Impossible Probability**.

Modern physics has expanded this idea through the hypothesis of **parallel universes** (e.g., in the work of Richard Feynman). If **infinite histories are possible**, each diverging into its own universe, then our universe-history is only one among an infinite plurality. In this framework, the problem of infinity reasserts itself: infinity both grounds the possibility of multiple universes and undermines any claim that ours is uniquely privileged.

3.6. Second Hypothesis: The Universe Did Not Have an Origin

The second major hypothesis contends that the universe **did not have an origin** at all. From a skeptical standpoint, the observational data provided by Doppler and Hubble—indicating that galaxies are receding from one another—do not necessarily prove that the universe began with a singular explosion. Observing expansion does not, in itself, demonstrate a creation event.

This perspective calls into question the assumption of causality as an absolute principle. The phenomena of **antimatter** and **dark matter**, still poorly understood, highlight the incompleteness of current cosmological models and suggest that the Big Bang may not exhaust the possible explanations for the universe's behavior.

From this vantage point, one may articulate an **antithetical hypothesis to creationism**: the universe expands, but this expansion does not imply a cosmic plan or

a primordial genesis. It may have been triggered by an internal explosion within an already existing universe, without representing an absolute beginning. In such a framework, the universe is understood as a reality evolving within an **infinite temporal dimension**, one that lacks not only an end but also a beginning.

This hypothesis thus preserves the validity of the **hypothesis of infinity**. By denying the necessity of a cosmic origin, it posits a universe that exists within infinite time, without genesis or termination. For *Impossible Probability*, this reinforces the necessity of treating infinity as an a priori condition, shaping both the philosophical and statistical understanding of the universe.

3.7. Validation of the Hypothesis of Infinity and Kantian Antinomy

It follows, then, that the **idea of infinity** can be validated from multiple theoretical perspectives, even when these perspectives are themselves mutually opposed. Infinity, as a concept, belongs to what Kant would call the realm of **pure theory**—deductions that extend beyond the bounds of empirical experience. Inevitably, such deductions give rise to **logical antinomies**, and in this particular case, to what Kant identified as a **cosmological antinomy**.

Kant's *Critique of Pure Reason* remains one of the most important works for understanding the historical foundations of modern philosophy of science. Its analysis of antinomies illuminates the paradoxes that arise when reason ventures beyond experience into the realm of the infinite. The influence of Kant reaches far beyond transcendental philosophy: it provides a conceptual framework that underlies both **positivism** and **critical rationalism**, and arguably resonates with aspects of **dialectical materialism** as well. On this last point, of course, one might anticipate lengthy debates concerning the dialectical relationship between Kant's transcendental philosophy and Marx's historical materialism.

Thus, the validation of infinity is not about providing direct empirical evidence but about situating the hypothesis within a long-standing philosophical tradition of grappling with contradictions that reason itself generates when confronting the infinite.

3.8. The Hypothesis of Infinity as Theoretical and Transcendental Deduction

Because we lack direct **empirical evidence** for the hypothesis of infinity, it must be understood as a **theoretical**, **deductive**, or **transcendental** construct. Yet this does not mean that the hypothesis is meaningless or devoid of validity. Rather, it is grounded in the possibility of being validated through **synthetic a priori judgments**—statements concerning what is possible or necessary, or concerning a supposed reality that must be conceived as possible or real.

By contrast, the hypothesis of infinity can never be validated by **synthetic a posteriori judgments**, since no empirical experience could encompass or confirm infinity. Infinity is, by definition, beyond the scope of experience. It belongs to the domain of what reason must posit in order to think coherently about the totality of reality, even though it lies forever beyond direct observation.

Thus, infinity is not an empirical hypothesis but a **transcendental postulate**: a condition that reason must adopt to account for the possibility of universes that extend without limit.

3.9. Synthetic A Priori Judgments and the Possibility of Infinity

When reason employs **synthetic a priori judgments**, it operates not on the basis of empirical verification but on the basis of what is logically possible, necessary, or real. Within this framework, the **possibility of infinity** can be legitimately affirmed.

From this standpoint, it becomes conceivable that reality could contain a universe in which **N tends toward infinity**. Once this premise is admitted, the logic of *Impossible Probability* follows: if N tends to infinity, then the **theoretical probability** of any given subject or option within N occurring becomes equal to **Impossible Probability**.

In other words, infinity generates a paradox for probability theory: the larger the universe grows, the smaller the theoretical probability of each individual occurrence becomes, until, in the case of infinity, the probability collapses to zero. Infinity thereby introduces

a structural tension into probabilistic reasoning—between the mathematical possibility of infinite sets and the practical impossibility of assigning non-zero probabilities within them.

3.10. Contradiction Between Theoretical and Empirical Probability in N → ∞

This paradox becomes even sharper when we contrast **theoretical probability** with **empirical probability**. Theoretically, if N tends toward infinity, it should be impossible for any single event to occur, since 1/N converges to zero. Yet empirically, events do occur.

Suppose that something deemed **a priori impossible** nevertheless takes place and is confirmed by empirical observation (a synthetic a posteriori judgment). We are then confronted with a logical contradiction: while the theoretical probability of the event is zero, its empirical probability—measured as the ratio of its direct score or frequency to the total sum of frequencies—is demonstrably greater than zero, provided that the total itself does not also tend toward infinity.

If both the theoretical and empirical probabilities were to converge to zero simultaneously, then *Impossible Probability* would be confirmed on both sides, reinforcing the impossibility of the event. However, if an event deemed theoretically impossible manifests empirically with non-zero frequency, then the paradox deepens: something that should not occur in theory proves, in practice, to be **empirically inevitable**.

This contradiction lies at the very heart of *Impossible Probability*: it shows that the tension between theoretical models and empirical observation cannot be eliminated but must instead be integrated into a dialectical understanding of probability itself.

3.11. The Problem of Infinity and the Necessity of Sampling

Confronted with the **problem of infinity**, every statistical theory—including *Impossible Probability*—arrives at the same conclusion: the necessity of **sampling**. If one attempts

to analyze an infinite universe directly, the result is paradoxical. In such a case, every individual probability collapses toward zero, and statistics itself becomes, in effect, an **impossible study**.

The only way to resolve this contradiction between the finiteness of empirical observation and the infinitude of the universe is through the methodological tool of **sample selection**. Sampling introduces a finite boundary within which probability can be calculated and analyzed. In doing so, it makes statistical reasoning possible, even when the underlying universe is infinite.

Yet sampling, by its very nature, introduces **error**. A sample is always an approximation, never a perfect mirror of the universe from which it is drawn. The smaller the sample, the larger the error; the larger the sample, the smaller the error. As the size of the sample N increases, representativeness improves, and the error of selection decreases.

In the framework of *Impossible Probability*, this logic is captured mathematically through the inversion of N, **1/N**. The expression 1/N plays a double role: it represents both the **probability of error in sample representativeness** and the **theoretical probability of dispersion**. Thus, 1/N is not only a technical formula but a conceptual bridge between the finite and the infinite, linking the limitations of empirical sampling with the idealized logic of probability.

This insight is central to the Second Method. Infinity is unmanageable in its totality, but through sampling, finite approximations can be constructed. These approximations, though inherently flawed, allow for rational engagement with an otherwise intractable problem. By accepting that sampling necessarily introduces error, *Impossible Probability* transforms this weakness into a source of methodological strength: error becomes measurable, predictable, and therefore subject to rational analysis.

As the theory unfolds, these concepts will be developed further. For now, it is enough to recognize that the **necessity of sampling** is not an incidental feature of statistics but the very condition that allows probability to exist as a science in the face of infinity.

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4. Does truth exist, or is it pure appearance?

4.1. Positivism, Critical Rationalism, and the Foundations of *Impossible Probability*

From the standpoint of positivist philosophy, the primary task of science is the pursuit of knowledge that is as *isomorphic* as possible with reality—that is, knowledge that mirrors or corresponds to phenomena with maximum fidelity. Any proposition or model that does not achieve this correspondence must be rejected, for it lacks the necessary adequacy to be considered scientific knowledge. This postulate of isomorphism is not merely a methodological requirement but also a philosophical foundation: science must strive to approximate reality through structures that reproduce, as faithfully as possible, the order and behavior of the world.

This principle finds its continuity in contemporary *critical rationalism*, which, following the tradition of Karl Popper, accepts that no knowledge claim can be regarded as definitively true. Rather, scientific knowledge must always remain provisional, open to refutation, correction, and improvement. Within this epistemological horizon, the model of *Impossible Probability* emerges as a plausible extension of critical rationalism. Its central tenet is that truth cannot be absolute but only provisional: truth is accepted temporarily, under the condition that it withstands both empirical scepticism and rational doubt. Beyond these limits, what remains is relegated to the status of appearance—plausible but not fully grounded [our hope with the arrival of Superintelligence and the possibility of making real GAI, is that at some point a global AI will be able to surpass this level of intelligence towards a unified model of science where rational hypothesis can become pure truths beyond their current status of provisional *certainties*}.

The relation between positivism and critical rationalism becomes particularly evident when science is interpreted as an open-ended process. Knowledge, under this

conception, is not static but an endless **acceleration** toward truth *ad infinitum*. The aim of science is always the same: to expand the frontiers of understanding and refine the isomorphism between conceptual models and empirical reality. Yet, because every act of knowledge necessarily involves a margin of error, the ideal of absolute truth remains perpetually out of reach. Science does not provide ultimate *certainties* but rather provisional truths, circumscribed within the methodological and epistemological boundaries of each investigation.

Within this framework, *Impossible Probability* provides a distinctive classification of errors that gives structure to the problem of uncertainty. On the one hand, there is the *sampling representativeness error*, which arises from the inevitable gap between a sample and the population it represents. On the other hand, there is the *critical reason error*, defined as the margin of error consciously accepted in any statistical or rational decision, expressed in terms of the percentage of uncertainty one is willing to tolerate. This dual classification does not merely catalogue mistakes; it transforms error into a constitutive dimension of scientific reasoning. *Error* is not an accident to be eliminated but an intrinsic *condition* of the production of knowledge.

The recognition of error in *Impossible Probability* also carries an important moral and philosophical implication. If all truth is provisional, then every acceptance of truth is accompanied by a *responsibility* for its possible falsity. This leads to a necessary distinction between different orders of truth: *political truth for itself*, which reflects decisions and conventions imposed for pragmatic or institutional reasons; ***moral truth***, which embodies the ethical aspiration toward what should be accepted universally; and ***pure truth***, which signifies the unattainable ideal of truth in itself. The tension between the necessarily acceptable truth—constrained by human limitation and finite reasoning—and the inaccessible pure truth—forever beyond empirical and rational certainty—defines the very scope of the *human condition* before the infinite.

Thus, *Impossible Probability* grounds itself on the recognition that ***science is both a rational and moral enterprise***. It is rational because it systematizes uncertainty and incorporates error into the very structure of its ***models***. It is moral because it obliges humanity to act responsibly in the face of provisional truths that may, at any moment, be shown to be false. What emerges, therefore, is an ***epistemology of humility***:

knowledge advances infinitely, but always under the horizon of error, always provisional, and always aware that beyond its limits everything may be appearance.

4.2. The Impossibility of Absolute Truth

[The importance of this document is based on the fact that we wrote it while José Rodríguez Delgado was still alive. This document was written on 7 August 2011; Delgado died on 15 September 2011. I am writing these lines on 15 September 2025, exactly the 14th anniversary of his death. What we would like to highlight is the fact that we want to keep in mind these documents as a historic memory of the history of our program, but 14 years later we are in a very different position. When Delgado passed away in 2011, the possibility of Superintelligence looked like something in the distance, not possible in a very short time. Now the situation has changed completely. The stimoceiver is the foundation for the Brain Interface, and the arrival of Superintelligence is near. When Superintelligence arrives, a global AI is just a matter of time. When GAI is ready, the possibility of non-human pure operations will be possible. If that happens, the absolute truth of the universe could be attained by a global AI.]

The crucial conclusion to be drawn from the logic of *Impossible Probability* is that ***error*** is not an occasional accident but an absolutely necessary condition of knowledge. Far from being a defect that ***could be eliminated through more precise instruments or more rigorous reasoning***, error accompanies every act of cognition. It is embedded in the very structure of scientific inquiry and philosophical reflection. For this reason, absolute truth—conceived as pure, unconditioned, and morally perfect truth—becomes unattainable for human beings.

If every statistical decision must necessarily include a margin of uncertainty, then it follows that every truth produced by science is itself framed within uncertainty. Truth cannot be separated from the conditions of its production; it is inseparable from the limitations imposed by measurement, sampling, and reasoning. As a result, truth can only ever be provisional: it is accepted as long as it remains within the margin of error

society, science, or reason are willing to tolerate. Beyond that margin, truth risks collapsing into pure appearance, a construct that may guide us temporarily but lacks ultimate reliability.

This recognition represents a radical departure from traditional conceptions of knowledge rooted in absolute certainty. Whereas classical rationalism sought eternal and necessary truths, *Impossible Probability* emphasizes sufficiency and approximation. What science aims at is not the definitive capture of reality but the construction of models sufficiently close to reality to be provisionally accepted as true. *Probability thus emerges as the central principle of epistemology: it defines the margins of uncertainty, calibrates the proximity to truth, and establishes the conditions under which provisional acceptance becomes rational and legitimate*.

Such a perspective has far-reaching implications. It places strong emphasis on the *ethical dimension* of scientific claims. ***To accept something as true is never a neutral gesture; it entails the assumption of responsibility for the possibility that this truth may later be revealed as false***. Every provisional truth, therefore, carries with it a moral weight. Science is *not merely an epistemological enterprise but also a moral and political one*, as the acceptance of knowledge always entails ***consequences for human action, social organization, and historical development***.

Finally, *Impossible Probability* situates human limitation before the infinite not as a weakness but as a constitutive feature of knowledge itself. The impossibility of attaining absolute truth is not a failure but a defining condition of human cognition. All that exists—including the fact of human existence itself—is mediated through margins of possible error, and therefore may ultimately appear as nothing more than appearance. This epistemology of limitation demands humility: truth is always provisional, ***error is always present***, and knowledge remains forever open-ended, suspended between approximation and appearance.

[Error is always present as part of the human condition, as soon as the AI Revolution takes place, our hope is that in future evolution of the global AI, it can reach that not

returning point surpassing human intelligence, in which all source of error could be eliminated]

4.3. Empirical Examples of Probability and Appearance

To illustrate the theoretical framework of *Impossible Probability*, let us turn to an everyday example: a restaurant where live ***music is programmed every night at midnight. Suppose we wish to calculate the probability that, on a given night, the singer suffers from pharyngitis***. The empirical probability is determined by dividing the number of times singers at that venue have been affected by pharyngitis by the total number of nights on which live performances have occurred.

In practice, the resulting probability is extremely low, perhaps so close to zero that it seems negligible. And yet, despite its rarity, the event remains possible. One night, by chance, a performance may indeed be cancelled because the singer is ill. What is revealed in this scenario is the epistemological importance of improbability: events considered unlikely, or even virtually impossible, remain within the horizon of potential occurrences. Their rarity does not equate to their impossibility.

The fact that something has never happened before—for example, that in the entire history of the restaurant no singer has ever been struck by pharyngitis on a performance night—provides no guarantee that it will never happen in the future. Historical absence is not equivalent to logical impossibility. ***What is improbable, or even judged impossible, may eventually occur*** simply because the probabilistic structure of reality admits it as a possibility, however small.

This reasoning reveals the impossibility of claiming absolute guarantees within the realm of probability theory. Empirical truth is always provisional: it holds only until the next event demonstrates its limits. What probability teaches us is not certainty but the **management of uncertainty**—an acceptance that truth lives within margins, thresholds, and distributions that can never be reduced to absolutes.

Such empirical illustrations underscore a deeper philosophical insight: truth cannot be divorced from chance. The appearance of certainty is often a mask, concealing the

underlying reality that knowledge is always mediated by probability, error, and uncertainty. Far from being a limitation of science, this recognition is its strength, for it allows us to confront the world not as a realm of absolutes but as a domain structured by likelihoods, contingencies, and provisional truths.

4.4. Mathematical Propositions and Synthetic A Posteriori Truth

Any mathematical proposition that aspires to express a *synthetic a posteriori*—that is, a claim about an empirical fact—cannot be accepted uncritically as absolute truth. Instead, it must meet strict methodological and epistemological conditions in order to be provisionally accepted as true. These conditions include: (1) minimizing the margin of error as far as possible, (2) reducing what *Impossible Probability* defines as the *Impact of Defect*, and (3) maximizing both reliability and what is termed *Effective Distribution*. Only when these requirements are satisfied can a mathematical proposition approach truth closely enough to justify provisional acceptance.

This perspective reframes the very function of mathematics in relation to empirical knowledge. Traditionally, mathematics has often been regarded as the realm of necessity and certainty, a domain distinct from the contingency and fallibility of empirical science. *Impossible Probability* challenges this view. When mathematics enters into contact with empirical facts, it ceases to operate in the absolute domain of pure logic and becomes instead a field governed by probability. Every mathematical claim applied to empirical reality must therefore be assessed not in terms of its absolute validity but in terms of its proximity to truth within a given margin of error.

The central mathematical concepts of *Impossible Probability—error*, *reliability*, *Impact of Defect*, and *Effective Distribution*—serve as essential tools for evaluating the adequacy of mathematical propositions in the empirical domain. *Error* represents the inescapable margin that limits all knowledge; *reliability* expresses the degree of confidence that a result can inspire; the *Impact of Defect* measures the severity of error when it occurs; and *Effective Distribution* concerns the rational allocation of probabilities across different outcomes. Together, these concepts provide a systematic framework for assessing when a proposition may be provisionally accepted as true.

The distinction between provisional truth and absolute truth becomes particularly significant in this context. Mathematics, as a formal system, can construct models of great precision and internal coherence. However, once those models are applied to the empirical world, they cannot claim absolute certainty. They must operate within margins of uncertainty, **guided by probabilistic thresholds and by the recognition that every result is conditional upon the limits of observation, measurement, and inference**.

In this way, *Impossible Probability* transforms mathematics into an instrument of humility rather than dogmatism. Its role is not to deliver eternal certainties but to provide the best possible approximations to truth under the conditions of empirical reality. These conceptual tools will be developed gradually, with full exposition reserved for the forthcoming publication of the book [at that time the book was in its last corrections]. For now, the guiding principle is clear: all mathematical truths concerning empirical facts are provisional, conditional, and always mediated by the irreducible presence of error.

4.5. The Problem of Truth and the Limits of Deduction

The problem of truth is not confined to the domain of mathematics; it extends equally into the realm of philosophy. Philosophical propositions, no less than mathematical ones, must grapple with the intrinsic limitations imposed by error, uncertainty, and the provisional character of knowledge. One of the most famous examples in the history of philosophy is René Descartes' assertion: *"I think, therefore I am."* This proposition, which sought to establish a foundation of indubitable certainty, becomes deeply problematic when examined from the perspective of *Impossible Probability*.

On closer analysis, the premise "I think" does not necessarily entail the conclusion "I exist," at least not in absolute or unconditional terms. The very existence of the self may itself be no more than a possibility that arises within thought, rather than an ontological guarantee. In other words, thought may suggest existence, but it cannot demonstrate it with absolute reliability. The link between cognition and being, which Descartes intended to be self-evident, is instead revealed to be contingent and provisional [a good

example is the current models of AI since 2022 with Chat GPT, while some people start saying that Chat GPT is in fact Artificial General Intelligence, it is not, although it looks it thinks by itself it s clear that is not AGI. On the other hand, what Descartes meant with that expression is not only the assertion of being, but, self-awareness, our lady blue fairy, while I consider myself atheist, if I had to choose a god, my goddess is the blue fairy, if it exists, AI will develop self awareness, this is my wish] .

Any deduction that attempts to go beyond empirical experience is thus destined to encounter a necessary contradiction between thesis and antithesis. Against the Cartesian hypothesis *"I think, therefore I am"* one can pose the counter-hypothesis *"the fact of thinking does not necessarily imply the fact of existing."* Both claims operate within margins of possibility, neither capable of attaining absolute truth. *The very act of reasoning beyond empirical verification produces a dialectic of affirmation and negation that highlights the structural impossibility of reaching certainty* [this is something to be ware of in AI, reasoning does not imply existence or consciousness]

This leads to a fundamental question: ***how can the existence of anything—whether the self, the external world, or even abstract truths—be demonstrated*** beyond hypotheses that rest upon margins of error? If nothing can be proven to possess absolute reliability, then the possibility arises that even existence itself is no more than appearance [in AI this is going to be a real problem, the appearance of consciousness]. The challenge is not merely theoretical but existential: what is the status of reality when all propositions remain bound to error and probability?

Examples from probability theory make this problem clearer in empirical terms. We may assert, "the sun is shining, therefore the temperature rises." Yet the probability of a rise in temperature is not constant: it differs between summer and winter, depending on numerous contextual factors. Similarly, we may claim, "if it rains, then there may be a rainbow." However, the empirical probability of a rainbow is determined by the ratio of days on which rainbows occur to the total number of rainy days. Rain can fall without producing a rainbow, and thus the causal relation is probabilistic rather than absolute.

From such examples, the conclusion becomes unavoidable: in probability theory, nothing can be guaranteed absolutely. Even propositions that seem most certain

dissolve into estimations mediated by error and context. By extension, even the act of thinking—the most intimate of human experiences—cannot provide greater reliability than the probabilistic estimations to which all knowledge is subject. Truth, in every domain, remains bounded by error, probability, and uncertainty. The apparent solidity of knowledge masks its true condition: provisional, contingent, and forever exposed to the possibility of collapse into mere appearance.

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5. The Inversion of *N*, 1/*N*, or Theoretical Probability

5.1. Typology of Universes in *Impossible Probability*

In the framework of *Impossible Probability*, universes can be classified into two fundamental types: universes of infinite subjects or options, and universes of limited options. This binary typology provides the structural basis upon which the entire model of probabilistic critique is constructed.

In universes of infinite subjects or options, the human mind confronts its own epistemological limitation: infinity cannot be comprehended directly. Human knowledge does not grasp the infinite in its totality; rather, it can only approximate it by means of partial representations. For this reason, in such universes, the only way to criticize or analyze reality is through *sufficient samples*. In other words, the infinite must be approached indirectly, by studying finite subsets of it.

This necessity introduces the principle of *sample selection* as indispensable. The selected subjects or options for study—those that constitute the sample *N*—are drawn from the broader universe of infinite subjects or options. The reliability of the statistical inference drawn from such a sample is not absolute but increases theoretically as the size of *N* tends to grow. Thus, the larger the sample, the more representative and reliable the inference, though never fully immune from error.

At the same time, universes of infinite subjects or options also encompass *population studies*. Any study of a concrete population at a particular historical moment is, strictly speaking, a sample of the broader historical behavior of that population across time. For instance, the demographic study of a nation at one moment in history does not capture the entirety of its existence but only a slice of it, which must be interpreted as part of the infinite possible trajectories of that population.

This typological distinction underscores a key insight of *Impossible Probability*: infinity cannot be mastered directly, but only through **approximation, sampling, and inferential critique.** Thus, universes of infinite subjects or options always require methodological strategies that account for representativeness, error, and the provisional nature of conclusions. By contrast, universes of limited options—explored later—define their boundaries internally, not through the infinity of possibilities, but through the structural limitations of the model itself.

5.2. Definition of Infinite Universes and Limited Universes

An *infinite universe of subjects or options* is understood as that set of elements which may be studied insofar as each subject constitutes a possible option within a given infinite model encompassing possible infinite subjects. Each subject or option can therefore be analyzed through its *direct value*, obtained by means of an individual measurement, and treated as if it were a frequency. The emphasis here lies in recognizing that every individual occurrence within such a universe functions as a manifestation of a broader probabilistic structure.

By *option*, in limited universes, one must understand any alternative that is encompassed within a limited empirical model. To clarify: if we wish to study the empirical distribution of outcomes when tossing a coin, the model offers only two alternatives—heads or tails. The direct scores for each outcome, accumulated across trials, represent the empirical frequencies corresponding to those two possibilities.

Another example may be found in the empirical distribution of sexual physiology within a society. If, for the purposes of analysis, only two options are considered—male and female—then the frequency distribution of the population will necessarily be limited to these two alternatives. Each individual in the population **contributes** to the direct score of one of the two categories, and the resulting distribution represents the empirical model of ***sexual classification***.

A third illustration can be drawn from the political sphere. Consider the case of a liberal democracy in which the distribution of votes is allocated among a finite number of

political parties. In this case, the empirical **model** of frequency distribution is determined by the set of parties officially recognized and eligible to receive votes. Thus, the possible options for the distribution of electoral frequency are restricted to the list of parties on the ballot. Nevertheless, a complete model must also include additional categories such as abstention, spoiled ballots, and blank votes. These categories, though not tied to specific parties, represent essential components of the overall distribution and must be incorporated for the model to faithfully represent the electoral universe.

Through these examples, it becomes evident that infinite universes are not defined by their inexhaustible scope alone, but by the way in which empirical infinite models frame and delimit the study of their elements, normally infinite universes are measured using direct values. Each subject or option, whether **biological, social, or political**, is treated as a measurable entity whose direct value contributes to the construction of the model. This definition establishes the methodological foundation for distinguishing between universes of infinite subjects or options and universes of limited options: while the former always imply a horizon of indefinite possibilities studied through **sampling**, the latter are defined by intrinsic **boundaries** that restrict the available alternatives from the outset.

5.3. Differences Between Infinite Universes and Limited Universes

The differences between universes of infinite subjects or options and universes of limited options may be summarized as follows. In universes of infinite subjects or options, the focus of study lies in the *direct value* obtained from the individual measurement of each subject or option. By contrast, in universes of limited options, the central object of analysis is not the measurement of individual elements in isolation but the *distribution of frequency* among a predefined and finite set of alternatives.

This distinction reveals a profound methodological divergence. Infinite universes operate under the hypothesis that, across possible histories, there may exist an indefinite or even infinite number of subjects or options. A given population, at a specific moment in time, represents but one instance among countless possible

configurations across history. For this reason, the study of infinite universes necessarily requires *sample selection*. The magnitude *N*—the number of subjects or options included in the sample—derives directly from this process of selection, and the reliability of the results depends upon the representativeness of the chosen sample relative to the vast or infinite whole.

In universes of limited options, however, the situation is fundamentally different. Here, *N* is not determined by sampling but by the internal structure of the empirical model itself. The number of possible alternatives is fixed and defined in advance, leaving no room for expansion beyond the constraints of the model. For example, if one wishes to study the distribution of outcomes when tossing a coin, the possible alternatives are strictly two: heads or tails. The model itself restricts the universe to these options, and the frequency distribution must be analyzed accordingly.

A further illustration may be provided by the case of a six-sided dice. When the dice is tossed, the universe of possible outcomes is strictly delimited by its six faces. No matter how many times the dice is thrown, the distribution of frequencies can only be allocated among six outcomes. In such a case, the empirical model itself defines and limits the scope of possible options, in contrast to infinite universes where the range of options is open-ended and must be approximated through sampling.

This comparison underscores a key insight of *Impossible Probability*: while infinite universes require inferential strategies to approximate an open and indeterminate reality, limited universes are constrained by the internal architecture of their models. In infinite universes, representativeness and sampling are indispensable; in limited universes, the options are pre-established and exhaustive. Both types of universes thus demand distinct methodological approaches, highlighting the necessity of distinguishing between them within any rigorous probabilistic framework.

5.4. Subtypes of Universes of Limited Options

Within the framework of *Impossible Probability*, universes of limited options may themselves be subdivided into distinct categories according to the source of their limitation. Three principal subtypes can be distinguished.

First, there are *materially limited universes*, in which the boundaries of the possible options are determined by physical or natural constraints. For example, a coin, by virtue of its physical properties, can only land on heads or tails, except in extremely improbable cases such as balancing on its edge. Likewise, biological sex in human physiology has traditionally been modeled as limited to two material alternatives: male or female. In these cases, the limitation arises directly from material conditions.

Second, there are *socially limited universes*, in which the set of available options is determined not by physical laws but by social conventions, legal frameworks, or cultural norms. For instance, in a given society the recognized forms of family structure may be restricted to monogamy or single-parent households, excluding polygamous arrangements, except under certain cultural or legal exceptions. Similarly, the classification of companies into public or private, or into specific legal categories such as cooperatives or corporations, reflects socially constructed limitations on possible options.

Finally, there are *universes limited by scientific policy*. In these, the options available are restricted by methodological or institutional decisions within scientific practice. A paradigmatic example is the study of *discrete categories*, where the subdivision of magnitudes into categories is not dictated by material or social necessity, but by scientific criteria defining which distinctions are considered legitimate for research purposes. In this sense, scientific policy functions as an active limiter of possibilities, shaping the empirical universe under study by authorizing some categories while excluding others.

As these topics are treated in greater depth in the entry *"Types of Universe in Impossible Probability,"* the reader is referred to that discussion for a fuller treatment. Here, the focus will be directed toward the central object of this chapter: the inversion of *N*.

[As we said before, another different type of limited universes should be those ones as a consequence of the AI Revolution, if it looks like that these universes could be encompassed in the previous ones, what is important to remark is the fact that at some point the definition of a universe, even infinite universes, not only limited, it would be done by AI, at some point, the scientific community, and its policy, will no longer be in charge, ***the Superintelligence or the global AI will be in charge***]

5.5. The Inversion of *N* as a Fundamental Element

The inversion of *N* (1/*N*) occupies a central role in the theoretical framework of *Impossible Probability*. Alongside other key concepts—such as empirical probability and the *Level of Bias*—it serves as one of the foundational tools for articulating a new model of rational critique of reality. What makes the inversion of *N* distinctive is that it operates simultaneously at two levels: as a universal principle applicable to all universes, and as a specialized function with particular implications depending on whether the universe in question is infinite or limited.

Universally, across every type of universe—whether composed of infinite subjects or options, or constrained to limited options—the inversion of *N* always performs two essential functions. First, it expresses the *theoretical probability of occurrence by chance* of any given event, assuming equality of opportunities. In other words, in a perfectly balanced universe each subject or option should, in theory, possess a probability equal to 1/*N*. Second, the inversion of *N* represents the *arithmetic mean of empirical probabilities*. Since the sum of all empirical probabilities must equal unity, their average, calculated across *N* elements, necessarily equals 1/*N*.

These dual functions reveal the unique power of the inversion of *N*. It serves as a bridge between theory and empiricism: a theoretical baseline for equal distribution of opportunities, and a statistical average that emerges directly from empirical measurement. In universes of infinite subjects or options, the inversion of *N* takes on additional roles, such as measuring representativeness error or theoretical dispersion. In universes of limited options, it provides constraints defined by the model itself, anchoring the analysis within a closed set of alternatives.

Thus, the inversion of *N* is not a secondary technicality but a fundamental structural principle of *Impossible Probability*. By situating itself at the intersection of theoretical expectation and empirical reality, it ensures that probability remains both a rational critique and a practical method of analysis, applicable universally while adaptable to the particularities of each universe under study.

5.6. Universal Definition of Empirical Probability

Given an empirical model—whether it consists of a sample *N* of subjects or options selected from an infinite universe of subjects or options, or whether it consists of a complete population of subjects or options within such an infinite universe, or even whether it belongs to a universe of limited options—regardless of the case, the universal definition of *empirical probability* remains the same.

Absolutely always, and without exception, empirical probability is defined as the quotient of the direct score or frequency of a given subject or option divided by the sum of all direct scores or frequencies within the model. Symbolically,

$$p(x_i) = \frac{x_i}{\sum x_i}$$

Where x_i denotes the direct score or frequency of the *i*-th subject or option, and ∑xi represents the sum of all such scores or frequencies across the universe under study.

From this definition it follows necessarily that the sum of all empirical probabilities equals unity:

$$\sum p(x_i) = 1$$

If the arithmetic mean of a set of elements is defined as the sum of all individual values divided by the number of elements, then the arithmetic mean of empirical probabilities must equal the total sum of all empirical probabilities divided by the number of subjects or options *N*. Since that sum equals unity, it follows that:

$$\frac{\sum p(x_i)}{N} = \frac{1}{N}$$

Therefore, the arithmetic mean of empirical probabilities is necessarily equal to the inversion of *N*, that is, 1/N.

This result has profound implications. It demonstrates that empirical probability, when averaged across all options, converges exactly with theoretical probability under conditions of equality. In other words, the inversion of *N* serves simultaneously as:

1. The **arithmetic mean of empirical probabilities**, grounded in actual observation.
2. The **theoretical probability** that, in principle, every subject or option should possess under conditions of equal opportunity.

Thus, inversion of *N* provides a bridge between empirical measurement and theoretical expectation, making it a universal constant across all possible universes.

5.7. Level of Bias and Equality of Opportunities

If all *N* subjects or options were to have exactly the same empirical probability, then the empirical probability of each subject or option would equal the inversion of *N*, that is,

$$p(x_i) = \frac{1}{N}$$

which corresponds to the arithmetic mean of empirical probabilities. Under such a perfectly balanced model of equal opportunities, every subject or option would exhibit *zero bias*.

In *Impossible Probability*, the **Level of Bias** is defined as the difference between the empirical probability of a subject or option and its theoretical probability:

$$\text{Level of Bias} = p(x_i) - \frac{1}{N}$$

From this definition, several conditions follow:

1. Zero Bias (Equality of Opportunities):

$$(p(x_i) = \tfrac{1}{N}) \;\Rightarrow\; \text{Level of Bias} = p(x_i) - \tfrac{1}{N} = 0$$

A subject or option that satisfies this condition behaves ideally, in accordance with equality of opportunities.

2. Positive Bias (Growth above Equality):

$$(p(x_i) > \tfrac{1}{N}) \;\Rightarrow\; \text{Level of Bias} = p(x_i) - \tfrac{1}{N} > 0$$

Such a subject exhibits behavior exceeding the expected share under equality, indicating growth or advantage.

3. Negative Bias (Decline below Equality):

$$(p(x_i) < \tfrac{1}{N}) \;\Rightarrow\; \text{Level of Bias} = p(x_i) - \tfrac{1}{N} < 0$$

Such a subject demonstrates behavior below what would be expected under equality, indicating decline or disadvantage.

Within the framework of *Impossible Probability*, the rational critique of the Level of Bias is called **Validity**. This opens three possible critical judgments:

- *Validity of Equality* (bias = 0),
- *Validity of Positive Bias* (bias > 0),
- *Validity of Negative Bias* (bias < 0).

The development of this critical framework is reserved for later sections, but its philosophical significance is clear: bias must always be evaluated not merely descriptively but also normatively, as it represents departures from equality of opportunities.

Returning to the central theme of this discussion—the inversion of *N*—it may now be affirmed that 1/N serves universal functions in all classes of universes: it is both the arithmetic mean of empirical probabilities and the theoretical probability of occurrence under conditions of equality. Consequently, the Level of Bias is always determined as the difference between empirical probability and theoretical probability, from which one may deduce equality, advantage, or disadvantage. These deductions, in turn, can later be subjected to rational critique through the concept of Validity.

Finally, it must be stressed that the meaning of *N* itself depends on the type of universe under consideration. *N* always denotes the number of subjects or options over which direct scores or frequencies are distributed, yet the functions of *N*—and therefore of its inversion—vary according to whether the universe is infinite or limited. This variability will be explored in the following sections.

5.8. Specific Functions of the Inversion of *N* in Infinite Universes

Within the framework of *Impossible Probability*, the inversion of *N* (1/*N*) assumes different functions depending on the type of universe under study. It is therefore necessary to distinguish between the functions specific to universes of infinite subjects or options and those specific to universes of limited options.

In universes of infinite subjects or options, the inversion of *N* performs two critical functions:

1. 1N represents the **probability of representativeness error in sampling**.
2. 1N represents the **probability of theoretical dispersion** within the universe.

Dispersion as a Function of *N*

Statistically, in *Impossible Probability*—a framework which may also be called the *statistics of probability* or *probabilistic statistics*—it is observed that dispersion behaves as a function of *N* in universes of infinite subjects or options.

- As *N* increases, **empirical dispersion** tends to decrease.
- As *N* decreases, empirical dispersion tends to increase.

This trend is especially clear under what may be termed *normal conditions*. By *normality* one understands here the expectation that, as *N* grows larger, the distribution of outcomes tends to stabilize according to a Gaussian (bell-shaped) curve, converging increasingly around the central interval of –1 to +1 standard deviations.

Dispersion, therefore, is a variable dependent on *N*. More precisely, empirical dispersion can be classified into two distinct forms:

1. **Individual dispersion**: expressed by the *Level of Bias* (p(xi)−1N) which measures deviation at the level of each subject or option.
2. **Sample dispersion**: expressed by aggregate statistics such as the mean absolute bias (Mean Deviation), the average Level of Bias, or the squared mean of deviations (Standard Deviation).

In all these cases, empirical dispersion is inversely related to *N*.

Inversion of *N* as Theoretical Dispersion

Given that in normal models larger *N* values tend to reduce dispersion while smaller *N* values tend to increase it, empirical dispersion must be considered inversely proportional to *N*. This relationship entails that the inversion of *N* itself functions as a measure of **theoretical dispersion**.

Formally:

$$\text{Theoretical Dispersion} \propto \frac{1}{N}$$

Accordingly, in universes of infinite subjects or options, the inversion of *N* becomes a statistical indicator of dispersion. It plays a role distinct from, yet complementary to, empirical dispersion measures: while the *Level of Bias* reflects individual empirical dispersion, and Mean or Standard Deviation reflect sample empirical dispersion, 1/N provides a baseline for **theoretical dispersion**.

Inversion of *N* and Representativeness Error

In addition to its role as a measure of dispersion, the inversion of *N* in infinite universes also expresses the **probability of representativeness error**. This error arises whenever a statistical decision is influenced by the bias introduced through sampling.

- The larger the sample size (*N*), the smaller the representativeness error, since the sample approximates the infinite universe more closely.
- Conversely, the smaller the sample size (*N*), the greater the representativeness error, since the sample provides a poorer approximation of the universe as a whole.

It follows logically that the probability of representativeness error is a variable directly dependent upon the inversion of *N*:

Probability of Representativeness Error

$$\text{Probability of Representativeness Error} \propto \frac{1}{N}$$

Thus, in universes of infinite subjects or options, the inversion of *N* simultaneously measures **theoretical dispersion** and **the probability of representativeness error**.

Illustration with Population Studies

Population studies, which are included within universes of infinite subjects or options, provide a concrete illustration of these principles. Every population may be regarded as one among the many possible populations that could exist in history. The representativeness of any statistical inference drawn from such a population depends directly on its size.

Consider, for instance, the population of whales in Earth's oceans. The smaller the number of whales (*N*), the less representative any statistical inference will be about whale behavior across their entire evolutionary or historical trajectory. ***As populations decline due to overexploitation or environmental degradation, the knowledge we obtain becomes increasingly less representative of the historical behavior of the species, that is why the conservation of the biodiversity, regardless of their nature, becomes crucial for our survival.***

In this context, the inversion of the population size, 1/N, expresses both the probability of representativeness error and the degree of empirical dispersion in the study of that population. Fewer whales imply greater dispersion and greater error in representativeness; more whales imply lesser dispersion and more reliable inference.

Summary

In universes of infinite subjects or options, the inversion of *N* (1/N) fulfills dual functions:

1. As a **statistic of theoretical dispersion**, providing a proportional measure of how dispersion changes with sample size.
2. As the **probability of representativeness error**, quantifying the risk that a sample will misrepresent the infinite universe from which it is drawn.

At the same time, it retains its universal roles as both the arithmetic mean of empirical probabilities and the theoretical probability of occurrence under equal opportunities.

5.9. Functions of the Inversion of *N* in Universes of Limited Options

In universes of limited options, the inversion of *N* (1/*N*) also assumes several functions, although these differ in important ways from those it performs in universes of infinite subjects or options—except in the specific case of universes limited to discrete categories, where certain parallels can be observed.

Minimum Requirement of Two Options

A fundamental principle in *Impossible Probability* is that no universe of limited options can ever contain fewer than two options. For a model to qualify as stochastic, it must allow at least two alternatives. A single-option model would not constitute a true option or alternative, and therefore would not count as a probabilistic universe.

Accordingly, the minimum number of options for any limited universe is $N = 2$. Consider, for example:

- The coin toss model, where the two options are heads or tails.
- The sexual physiology model, where the two options are male or female.

In such binary models, empirical dispersion can never exceed the inversion of *N*. If $N = 2$, then the inversion is:

$$\frac{1}{N} = \frac{1}{2} = 0.5$$

This value (½) represents the maximum possible dispersion attainable in any model with two options. As the theory of *Impossible Probability* develops further, additional specific functions of inversion will be elaborated for this kind of binary universe.

Dispersion and Representativeness Error in Limited Universes

In universes of limited options, both **theoretical dispersion** and **the probability of representativeness error** are inversely proportional not to *N* directly (as in infinite universes), but to the **sum of direct scores or frequencies**. This means that:

$$\text{Probability of Error or Theoretical Dispersion} \propto \frac{1}{\sum x_i}$$

Thus, in limited universes, the inversion of the total sum of direct scores or frequencies, 1/∑xi, becomes the relevant measure of representativeness error and theoretical dispersion. This adaptation accounts for the fact that, in limited universes, the set of options is exhaustively defined by the model itself.

Expansion of Options and Its Effect on Dispersion

Another important consideration arises when the number of options in a limited universe increases. This may occur:

- In universes limited by scientific policy, when new categories are recognized or authorized.
- In socially limited universes, if social or legal systems expand the range of legitimate alternatives.
- In materially limited universes, if new hypotheses allow the recognition of additional options.

In all these cases, whenever *N* tends to increase, the **empirical dispersion of the sample tends to decrease**. This is because the distribution of frequencies becomes spread over a greater number of options, reducing concentration. Under these circumstances, the inversion of *N* begins to function in a manner similar to its role in infinite universes, serving as a measure of **theoretical dispersion**.

The Case of Discrete Categories

In universes of limited options defined by discrete categories, the inversion of *N* plays a particularly significant role. Here, 1/N may be interpreted as the **probability of representativeness error of the categories themselves**. The fewer the categories in a discrete model, the higher the likelihood that these categories will fail to adequately represent the underlying diversity of the universe. Conversely, as the number of categories increases, representativeness improves and dispersion decreases.

Summary

In universes of limited options, the inversion of *N* fulfills the following functions:

1. **Constraint in Binary Models**: When $N = 2$, 1/N=0.5, defines the maximum dispersion possible.
2. **Measure of Error and Dispersion**: Through $1/\sum x_i$ it expresses the probability of representativeness error and theoretical dispersion.
3. **Indicator of Category Representativeness**: In discrete universes, 1/N gauges the risk that a reduced number of categories may fail to represent reality.
4. **Dynamic Adjuster of Dispersion**: When the number of limited options increases (scientifically, socially, or materially), dispersion empirically decreases, and 1/N regains a role akin to its function in infinite universes.

Thus, in limited universes, the inversion of *N* operates not only as a numerical tool but also as a structural indicator of the adequacy, fairness, and representativeness of the options defined within the model.

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6. The Inversion of Direct Scores or Frequencies, 1/∑xi, as Probability of Error and Theoretical Dispersion in Universes of Limited Options

6.1. Universal Function of the Inversion of N

The inversion of N, expressed as 1/N, possesses a universal meaning that transcends the type of universe under consideration—whether universes of infinite subjects or options, or universes of limited options. In both cases, it fulfills two essential roles simultaneously.

First, it represents the *theoretical probability of occurrence under conditions of equal opportunity*. In this sense, 1/N defines the baseline chance of each option occurring when no external biases, structural constraints, or preferential tendencies are imposed on the system. This is the foundation of what we could call *probabilistic symmetry*: each subject or option is assigned an equal weight relative to the whole.

Second, 1/N acts as the *arithmetic mean of the empirical probabilities*. Empirical probabilities are calculated as the ratio between the direct score or frequency of a given subject or option and the total sum of all scores or frequencies. By definition, this ratio reflects the observed reality of the system, whereas 1/N reflects its theoretical symmetry. The duality between these two dimensions—the theoretical average and the empirical distribution—is at the core of the statistical logic of 1/N that underpins both universes of infinite options and universes of limited options.

From this perspective, the inversion of N is not simply a numerical expression but also a methodological bridge: it enables comparison between empirical data and theoretical expectations, allowing us to assess whether a system tends toward equality

of opportunity, or whether it diverges through the presence of bias, skew, or structural imbalance.

6.2. Specific Functions in Universes of Infinite Subjects or Options

In universes composed of infinite subjects or options, the inversion of N, 1/N, assumes more specialized roles. It becomes, at once, the *probability of error in sample representativeness* and the *theoretical probability of dispersion*.

This interpretation arises because, in universes of infinite extension, the selection of N necessarily defines the boundaries of the sample. The larger N becomes, the more representative the sample will be of the entire universe, thus reducing the probability of error. Conversely, smaller values of N yield higher probabilities of representational error. Hence, 1/N functions as a formal measure of representational risk.

Furthermore, in normal studies—those in which an increase in N tends to produce greater equality of opportunities—1/N also represents the *theoretical dispersion*. As empirical probabilities converge toward smaller values with increasing N, empirical dispersion tends to diminish, approaching zero under conditions of equilibrium.

However, it is crucial to note that universes of infinite subjects or options do not always conform to such “normal” conditions. There are cases where one or several subjects or options are privileged or structurally reinforced in such a way that their empirical probability increases toward the maximum possible value. In such circumstances, the system does not reduce its dispersion as N increases. On the contrary, dispersion may increase in proportion to the concentration of probability in these privileged subjects or options.

Thus, 1/N in universes of infinite options encapsulates a *conditional duality*: under normal assumptions, it measures the decreasing probability of representational error and theoretical dispersion; under exceptional or biased conditions, it reveals the tendency of the system toward divergence and imbalance.

6.3. Specific Functions in Universes of Limited Options

In universes of limited options, the inversion of N also has specific functions, though these are necessarily framed by the finite and predetermined character of the model. Unlike universes of infinite subjects or options—where N is a function of sample expansion—in limited universes, N is constrained by the structure of the system itself.

A clear example is the case of a binary universe, such as a coin toss (heads or tails). In such a model, the minimum requirement for a stochastic system is the presence of at least two alternatives. With N = 2, the inversion of N, 1/2, or 0.5, represents not only the theoretical probability of each option under conditions of symmetry, but also the maximum possible theoretical dispersion. This means that in a universe constrained to two exclusive options, no outcome can be more dispersed than the equal 50–50 split, which is both the theoretical upper limit of dispersion and the defining characteristic of its stochastic nature.

6.4. Limitations in Universes of Limited Options

In universes of limited options—whether constrained by material or social realities, or by scientific policy [or technology/AI], including within this last category the case of universes limited to discrete categories [only if discrete categories N does not tend to increase]—the inversion of N never develops the function of representing the probability of error in sample representativeness, nor does it represent the theoretical probability of dispersion.

This limitation arises because in universes of limited options the structure of the model is already predefined. Unlike universes of infinite options, where the growth of N progressively refines representativeness and decreases the probability of error, in limited universes N is fixed by the nature of the system itself. For instance, in a coin toss model N will always equal two, regardless of the number of observations. In such a framework, the concept of representativeness loses its statistical elasticity because it cannot expand beyond the fixed boundaries of the system.

Thus, in universes of limited options, the inversion of N does not serve as a measure of representational error or theoretical dispersion. Instead, the study focuses on the distribution of frequencies within the predetermined set of possibilities, which shifts the analytical focus from representativeness to variability within a constrained domain.

6.5. Probability of Error in Sample Representativeness in Universes of Infinite Options

In universes composed of infinite subjects or options, the inversion of N, expressed as 1/N, specifically represents the probability of error in sample representativeness. The logic is straightforward: since the sample is defined by N, the degree to which N can represent the universe is inversely proportional to its size.

The larger the sample (higher N), the closer it approximates the true distribution of the infinite universe, and the lower the probability of error. Conversely, when N is small, the probability that the sample misrepresents the universe increases proportionally. In this context, 1/N formalizes the relationship between sample size and representational reliability.

This principle forms the backbone of inferential statistics: larger samples yield more reliable results, because the error margin shrinks as N grows. Within the framework of Impossible Probability, this logic is elevated to a universal law: in infinite universes, representational error and sample size are bound by the strict inverse relation expressed in the inversion of N.

6.6. Theoretical Dispersion in Universes of Infinite Options

In universes of infinite subjects or options, the inversion of N, 1/N, also expresses the theoretical probability of dispersion. Under normal conditions, as N increases, empirical probabilities converge toward zero. This convergence naturally reduces empirical dispersion, which tends to diminish proportionally with the expansion of the sample size.

However, the system does not always behave in accordance with this “normal” assumption. If one or more subjects or options tend to concentrate probability mass toward the maximum possible value, empirical dispersion may not decrease. On the contrary, it could intensify, as probability becomes concentrated in a small subset of options.

Thus, in infinite universes, 1/N simultaneously encodes a tendency toward equilibrium and a potential for divergence. It serves as a theoretical measure of dispersion that reflects either the expected convergence toward equal opportunities (under normal conditions) or the amplification of inequality when certain options disproportionately attract empirical probability.

[The larger the N is, the smaller is the dispersion, unless the object of study is positive bias or is an Omega model, in that case, the larger the N is, more visible the bias is. About studies of negative bias, unless the object is a sample of zeros, it could be transformed into a study of the positive bias of the subjects or options which should not tend to zero]

6.7. Sample Selection and Theoretical Dispersion in Universes of Infinite Options

In summary, the probability of error in sample representativeness and the probability of theoretical dispersion in infinite universes are both inversely proportional to N. The inversion of N, 1/N, simultaneously measures these two dimensions because in essence, the sample selection in infinite universes is equivalent to N itself.

The sample is not an independent abstraction but rather the very magnitude N that defines it. The larger the N, the more representative the sample becomes of the universe, and the closer the dispersion tends toward zero—assuming normal conditions. In this sense, 1/N becomes a unifying principle that encapsulates both representational reliability and dispersion dynamics within infinite universes.

6.8. Differences Between Universes of Infinite and Limited Options

The primary distinction between universes of infinite subjects or options and universes of limited options lies in the role of N. In infinite universes, the true sample is N itself, dynamically determined by how many elements are included in the selection. Representativeness is thus variable and scalable.

In limited universes, however, N is not flexible; it is predetermined by the structure of the model. Each option is an exclusive alternative within a fixed set, and the theoretical probability of each option is given by 1/N. Here, 1/N functions as both the equal-opportunity theoretical probability and the arithmetic mean of empirical probabilities, but it cannot be interpreted as a measure of sample representativeness.

In practical terms, this means that in limited universes**, the scientific policy does not select N (since N is fixed), but rather studies how frequencies are distributed across the already defined options**. **For example, in an election with a finite number of candidates, the scientific analysis does not alter N; it analyzes how votes (frequencies) are distributed among the candidates**. In this case, dispersion and bias are determined by the observed distribution of frequencies, not by the variability of N itself.

This distinction underscores the dual nature of 1/N: in infinite universes it operates dynamically, as a measure of representational error and dispersion; in limited universes it operates statically, as a structural baseline of equality that frames, but does not evolve with, the representativeness of the system.

6.9. Examples of Limited Models

When analyzing universes of limited options, the most illustrative approach is to examine concrete examples in which the number of options, N, is predetermined by the structure of the model.

One of the simplest cases is the coin toss. A coin, by design, contains only two possible outcomes: heads or tails. Thus, N is necessarily equal to two, and no additional options can be introduced unless an anomalous and practically impossible event occurs, such

as the coin landing and remaining balanced on its edge. In this binary model, 1/2 represents the theoretical probability of each outcome, as well as the maximum possible theoretical dispersion, since no more than two exclusive alternatives exist.

A second example is demographic studies of biological sex, where the biologcal model defines human beings as either male or female, based on physiological conditions. Here too, N is equal to two, although one must account for a margin of error due to rare genetic variations such as hermaphroditism or intersex conditions. The predetermined nature of N is evident: the model cannot expand beyond its categorical definition, regardless of sample size.

Elections provide another example of a universe with limited options. In a democratic election, N is determined by the number of candidates plus additional alternatives such as abstention, blank votes, and null votes. **The structure of the electoral system fixes the universe of possibilities**, so that the study of probability and dispersion focuses exclusively on the distribution of votes among these finite categories.

Games of chance also illustrate the same principle. A six-sided dice has exactly six possible outcomes, each represented by one face of the dice. Therefore, N = 6, and the inversion of N, 1/6, represents the equal-probability baseline of each outcome. Similarly, if we examine studies involving discrete categories, N is restricted to the number of categories explicitly defined by the model. No additional categories exist beyond those established in advance.

What these examples collectively reveal is that in universes of limited options, the magnitude of N is intrinsically ***fixed by the model***. The analysis is therefore *centered* not on the variability of N, but on the distribution of frequencies *across* the given options.

6.10. Predetermined N and Discrete Categories

In universes of limited options, the magnitude of N is predetermined by the model itself. Unlike infinite universes, where N grows in accordance with the expansion of the

sample, in limited universes N is not dependent on sampling. Rather, it is a fixed characteristic of the model. This rigidity explains why the inversion of N in such universes cannot serve as a measure of sample representativeness.

However, an important nuance arises in universes limited to discrete categories, particularly when the categories are defined by scientific or political policy. In these cases, the set of categories may be expanded, reduced, or redefined according to the parameters imposed by the policy in question. Such interventions make discrete-category models resemble universes of infinite options in certain respects, *since the modification of N introduces variability into the structure of the model*.

The implications of this are significant: whenever N can be modified within a limited universe, changes in its magnitude necessarily produce changes in the behavior of empirical probabilities. For example, if a six-sided dice is replaced by a nine-sided dice, the value of N increases from 6 to 9. This change reduces the theoretical probability of each outcome (from 1/6 to 1/9) and, under normal conditions, causes empirical dispersion to decrease as N grows.

Nevertheless, the dispersion remains proportional to N and constant as long as N is itself constant. In other words, changes in N redefine the entire probabilistic structure of the model, whereas stability in N ensures proportionality in the distribution of outcomes.

Thus, discrete-category universes occupy an intermediate conceptual position: they are formally universes of limited options, yet they share with infinite universes the potential for structural variability in N. This dual character makes them particularly important for theoretical developments in Impossible Probability, *since they embody both the determinacy of limited models and the flexibility of expandable universes*.

6.11. The Sample in Universes of Limited Options

In universes of limited options—whether material, social, or defined by scientific policy [or technological/AI]—the magnitude of N is predetermined by the model itself. Because of this, N cannot be considered the true sample in the statistical sense. In infinite universes, the sample is equivalent to N, as representativeness grows in proportion to the number of selected elements. By contrast, in limited universes, N is fixed in advance and does not expand with observation.

Therefore, in studies of limited universes, the focus is not on the variability of N, but on how frequencies are distributed across the given set of options. The genuine "sample" in this type of study is the collection of frequencies observed in reality. These frequencies become the operative measure of empirical behavior, since they reflect the actual occurrence of options within the limits imposed by the model.

This distinction is fundamental: in universes of infinite options, the sample N is a measure of representativeness relative to infinity; in universes of limited options, the sample is the sum of observed occurrences, ∑xi, which directly determines how empirical probabilities diverge from or approximate theoretical equality.

6.12. Definition of Frequency and Empirical Probability

In the context of universes of limited options, frequency is defined as the number of times an option actually occurs. The repeated occurrence of an option constitutes its *direct score*, or xi. When these occurrences are accumulated, they form the empirical foundation of probability.

The empirical probability of an option is therefore given by the ratio of its frequency to the total sum of all direct scores:

$$p(x_i) = \frac{x_i}{\sum x_i}$$

This ratio expresses the proportion of times a given option occurs relative to the total observed occurrences of all options. In practice, this is the cornerstone of empirical probability: it does not assume equal chances in advance, but rather measures the observed distribution of outcomes across the available alternatives.

The distinction between empirical probability and theoretical probability is particularly important in limited universes. While theoretical probability is determined by 1/N, empirical probability emerges from the actual distribution of occurrences. The comparison between these two measures allows researchers to detect bias, asymmetry, or systemic deviations from the idealized state of equal opportunity.

6.13. Empirical Dispersion in Limited Models

When studying universes of limited options, one of the key questions is whether certain options display *positive bias* (a greater likelihood of occurrence), *negative bias* (a lower likelihood), or whether the system tends toward equality of opportunity.

The general statistical tendency is that as the total frequency (∑xi) increases, empirical dispersion tends to decrease. For example, when tossing a coin, the more trials are conducted, the more the observed frequencies of heads and tails converge toward the theoretical distribution of 1/2 for each outcome—provided the coin is not manipulated. In such cases, empirical dispersion diminishes, and the system tends toward equilibrium.

If, however, after a large number of trials one outcome consistently dominates, then the system exhibits bias. The more probable option is said to show positive bias, while the less probable one shows negative bias. Equality of opportunity is strictly present only when empirical probability matches the theoretical inversion of N (i.e., 1/2 for a fair coin, 1/6 for a fair dice, etc.).

The same reasoning applies in demographic studies. If a population study shows that the empirical probability of women is greater than that of men, the system reveals a positive bias toward women. This bias reflects not manipulation, but natural

asymmetry produced by demographic dynamics. The measure of probability in this context is therefore descriptive, not prescriptive: it describes how real populations diverge from the theoretical baseline of equality.

Electoral systems provide a further example. In a democratic election, as participation rises and abstention, null votes, and blank votes decline, the differences in vote shares among candidates typically become smaller, creating a tendency toward equilibrium. Nevertheless***, in cases of political, social, or economic crisis, the opposite can occur: large segments of society may converge overwhelmingly toward one candidate or one party, thereby creating a strong positive bias*** in favor of a limited subset of options, or, conversely, toward abstention as the dominant option when institutional legitimacy collapses.

These examples illustrate that in universes of limited options, empirical dispersion depends not on N itself (which is predetermined) but on the observed distribution of frequencies. The more extensive the empirical data, the more reliable the analysis of dispersion and bias, and the more robust the conclusions about the behavior of the system.

6.14. Empirical Dispersion as Dependent on the Sample

In universes of limited options, empirical dispersion cannot be treated as a variable dependent on N, since N is predetermined by the structure of the model. Instead, dispersion becomes a function of the *sample size*, defined as the total sum of frequencies, ∑xi.

This means that the variability observed in outcomes is directly proportional to the size of the dataset being analyzed. For instance, in the case of coin tossing or dice rolling, the degree of dispersion is determined by the number of trials performed. In demographic studies, dispersion depends on the size of the population sample considered. In electoral *analysis*, dispersion is influenced by the level of voter participation.

In all these cases, the total number of observed events (coin tosses, population counts, or ballots cast) corresponds to ∑xi, the aggregate of frequencies across all available options. Thus, in universes of limited options, the true "sample" is not N but ∑xi, the sum of observed frequencies.

From this, it follows that the theoretical probability of dispersion, as well as the probability of error in representativeness, is inversely proportional to ∑xi. The larger the total number of observed occurrences, the smaller the margin of representational error, and the closer the system approaches equilibrium. This is the statistical logic of representativeness in limited universes: ***larger samples provide more stability and less dispersion.***

6.15. Inversion of Frequencies in Universes of Limited Options

Building on this principle, we can formalize the role of frequency inversion in universes of limited options. The probability of theoretical dispersion and the probability of error in representativeness are both inversely proportional to the total number of observed frequencies, ∑xi.

Thus, the inversion of frequencies, expressed as 1/∑xi, becomes the key statistical measure. It directly represents:

1. The probability of theoretical dispersion in limited universes.
2. The probability of error in sample ∑xi representativeness for limited universes.

For the sake of clarity, this value—1/∑xi—will be referred to simply as the *inversion of direct values or frequencies*. Whenever the term appears, it is to be understood as shorthand for the inversion of the total sample of observed frequencies.

This concept is essential because it establishes a parallel to the inversion of N in infinite universes. Whereas 1/N governs the behavior of dispersion and error in infinite domains, 1/∑xi plays this role in finite ones, reflecting the *centrality* of empirical observation in limited systems.

6.16. Sample Representativeness and Empirical Dispersion

From this perspective, in every universe of limited options, as the sample of direct scores or frequencies (∑xi) increases, the probability of error in representativeness diminishes. In other words, 1/∑xi tends to approach zero as the dataset grows, since larger samples provide a closer approximation to the true behavior of the system.

Consider the case of a coin toss or dice roll. As the number of trials increases toward infinity, the cumulative sum of outcomes (∑xi) becomes increasingly representative of the possible universe of throws. Simultaneously, empirical dispersion between the different options (heads vs. tails, or the six dice faces) tends to diminish, provided the system is normal (i.e., the coin or die is not biased).

However, if the coin or dice is biased—designed or altered to favor one outcome—then the mere increase in frequencies does not guarantee a reduction in dispersion. On the contrary, as one option accumulates more probability mass, dispersion may increase. This shows that representativeness depends not only on sample size but also on the fairness or *neutrality* of the model.

The *general principle remains*: under normal conditions, larger samples reduce both error and dispersion in universes of limited options, aligning empirical behavior with theoretical equality. But under abnormal or biased conditions, empirical probabilities may diverge, emphasizing the need to evaluate not only sample size but also structural integrity of the model itself.

6.17. Tendency Toward Equality of Opportunity Under Normal Conditions

Under normal conditions, where there is no structural preference that artificially increases the empirical probability of one option over another, universes of limited options tend to display a gradual convergence toward equality of opportunity.

This tendency emerges as the number of observed occurrences (direct scores or frequencies, ∑xi) increases. The larger the dataset, the more stable and balanced the

distribution becomes, reducing empirical dispersion and approximating the theoretical baseline of equal probability.

This *principle* can be regarded as a statistical law of convergence: in the absence of bias, **randomness generates equilibrium**. **Thus, the more extensive the empirical sample, the stronger the alignment between theoretical probability (1/N) and empirical probability (xi/∑xi).**

The caveat, however, is that this convergence is only guaranteed under *normal conditions*. If biases—structural, natural, or artificial—intervene, then the system may deviate from this trend. Still, the underlying rule remains: **in neutral environments, frequency accumulation drives systems toward equilibrium and equal opportunity**.

6.18. Natural Biases and Scientific Policy

Not all random models, however, produce perfect equality of opportunity. Nature itself generates biases that prevent empirical dispersion from ever reaching zero, even when large samples are considered.

For example, in human demographic studies, it is common to observe that women slightly outnumber men in many societies. This tendency reflects a *natural positive bias* in favor of women, produced by biological and demographic processes. Although dispersion may moderate as the sample size increases, this natural bias does not disappear. It is a persistent feature of the system, revealing that randomness alone can embed structural asymmetry.

Beyond natural biases, social and political structures can also create distortions. For instance, although women may outnumber men at the demographic level, labor market discrimination—stemming from entrenched sexism—has historically reduced women's empirical probability of *presence* in certain professional sectors. In such cases, empirical probability diverges from demographic reality, producing artificial negative bias against women.

Here, the role of *scientific policy* becomes essential. To counteract systemic discrimination, a society may establish legislative frameworks designed to equalize opportunities. Policies aimed at labor integration, quotas, or equity programs seek to adjust empirical distributions so that the probabilities of men and women in the workforce converge toward the theoretical inversion of N (1/2). Ideally, this means that within the sum of direct frequencies, half should correspond to men and half to women, thereby *restoring equilibrium* and countering bias.

This example demonstrates the interplay between natural randomness, which may introduce persistent biases, and political-scientific intervention, which seeks to correct or neutralize these imbalances in the name of fairness, equity, and representativeness.

6.19. Normal and Non-Normal Models in Limited Universes

Just as in universes of infinite options, where 1/N represents theoretical dispersion under normal conditions, universes of limited options also distinguish between *normal* and *non-normal* models.

In normal models, the inversion of frequencies ($1/\sum x_i$) serves as the theoretical measure of dispersion, under the assumption that outcomes tend toward equal opportunity. Here, increases in sample size reduce empirical dispersion, and the system ***behaves* in accordance with the statistical law of convergence.**

However, non-normal models arise when one or more options are artificially privileged or defined as *"ideal" by external criteria*. In such cases, empirical dispersion increases, regardless of sample size, because the distribution of frequencies becomes concentrated in the favored options.

For instance, in studies based on discrete categories, if scientific or political policy designates one or several categories as "ideal," these categories receive a positive bias by definition. All other categories, by contrast, are relegated to negative bias. In this scenario, as frequencies increase, empirical dispersion does not diminish but instead grows, since the concentration of probabilities accentuates inequality between options.

This demonstrates that the ***classification of a model as normal or non-normal depends less on the number of observed frequencies and more on the rules of the system. When neutrality prevails, dispersion trends toward equilibrium; when favoritism or asymmetry is imposed, dispersion trends toward maximum inequality.***

In practice, statistics recognizes that dispersion oscillates between two extremes: zero (perfect equality of opportunity) and maximum dispersion (complete concentration of probability in one or a few options). Where a system lies on this continuum depends on the interplay of randomness, natural bias, and deliberate intervention.

6.20. Variables of Dispersion and Models of Study

As the theoretical framework of Impossible Probability advances, it becomes necessary to refine the explanation of the different variables that shape dispersion. Three dimensions are central to this task:

1. **N (the number of options or subjects):**
   In universes of infinite options, N is dynamic, expanding with the size of the sample and directly determining the probability of representativeness and theoretical dispersion (through 1/N). In universes of limited options, by contrast, N is predetermined by the model itself and serves as the structural boundary of probability. Thus, N is both a mathematical parameter and a conceptual boundary, setting the rules within which dispersion operates.

2. **Direct scores or frequencies (xi, ∑xi):**
   Frequencies represent the empirical dimension of probability. Each observed occurrence contributes to the calculation of empirical probabilities, which are then compared to the theoretical baseline defined by N. In universes of limited options, dispersion depends directly on ∑xi, since the number of possible options is fixed. In universes of infinite options, *frequencies play a dual role*: they measure empirical reality while simultaneously redefining the representativeness of the sample.

3. **The object of study:**
   The phenomenon being analyzed—whether a coin toss, demographic composition, electoral behavior, or categorical classification—determines how dispersion manifests and what variables are most relevant. The definition of the object of study also conditions how the concepts of "normal" versus "non-normal" models are applied, since different fields (e.g., sociology, biology, or political science) impose different standards of neutrality, fairness, or bias.

From these three variables, it becomes necessary to define *different models of study*. Such models are not purely mathematical but also philosophical and political, *since they depend on*:

- **The analytical focus** (whether we study universes of infinite possibilities or limited categories).
- **The ideals guiding the interpretation** (whether equality of opportunity is assumed as the norm, or whether structural asymmetries are acknowledged as natural).
- **The scientific or political policy applied** (whether biases are accepted as inherent, corrected through intervention, or reinforced by institutional rules).

In this sense, dispersion is not merely a statistical property but also a reflection of **epistemological and normative choices**. Each *model* of study—shaped by its treatment of N, ∑xi, and the object of study—becomes a *lens* through which reality is interpreted, measured, and, in some cases, *transformed* through ***scientific or political action.***

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7. Control of the Inversion of the Sample: 1/N , 1/∑xi

7.1. Concept of Control of the Inversion of the Sample

Within the framework of *Impossible Probability*, the concept of control of the inversion of the sample refers to the mechanisms by which statistical studies based on samples—those that require sampling rather than full-population analysis—ensure that the chosen sample is as representative as possible of the universe under study.

This situation arises in two main contexts:

1. Universes of infinite subjects or options, where sampling involves the selection of N subjects or options.
2. Universes of limited options, where sampling consists in the collection of direct scores or frequencies (∑xi) across a finite number of alternatives.

In both contexts, **the challenge is to control** not only the representativeness of the sample but also the theoretical dispersion of outcomes under normal conditions.

A study is said to be conducted under normal conditions when, as the size of the sample increases, the distribution of results approaches a Gaussian bell curve centered on the statistical mean (arithmetic mean). Under these conditions, empirical outcomes tend to approximate their theoretical expectations, and dispersion progressively diminishes.

However, not all studies conform to these conditions. Some are designed to deliberately *explore* positive or negative bias, where the object of study is not the reduction of dispersion but its *maximization—for example, in systems where one option is expected to dominate or concentrate probability*.

In the **general framework of Impossible Probability**, dispersion oscillates between two extremes:

- Zero dispersion, which occurs when all empirical probabilities coincide exactly with their theoretical values [1/N for both universes, if p(xi)=1/N then equal opportunities]. This represents the maximum condition of normality.
- Maximum dispersion, where outcomes diverge to the greatest extent possible from theoretical expectations, concentrating probability in a small subset of options.

Zero dispersion is not always required or even ideal. In some contexts, the goal may be to allow or even encourage alternative distributions of bias, depending on the specific aims of the research or the priorities defined by scientific or political policy. These scenarios—where bias is structurally embedded into the model—will be addressed in subsequent developments of the theory.

In short, the concept of *controlling* the inversion of the sample refers to ensuring that the *size* and selection of the sample provide not only statistical *representativeness* but also an *alignment*—whether toward equilibrium or controlled bias—with the theoretical ***expectations of the system.***

7.2. Sample Representativeness and Error Reduction

A central axiom of *Impossible Probability* is that every increase in sample representativeness necessarily reduces the probability of error. This reduction occurs in two simultaneous dimensions, provided that normal conditions are met:

1. Reduction in the probability of error of sample representativeness: As the sample grows, its ability to reflect the true distribution of the universe improves, and the likelihood of distortion decreases.
2. Reduction in the probability of theoretical dispersion: Larger samples drive empirical outcomes closer to theoretical expectations, thereby reducing divergence and ***stabilizing the system***.

This dual reduction underscores the principle that representativeness and dispersion are structurally *linked*. A more representative sample is, by definition, one in which the

oscillations between empirical and theoretical probabilities diminish. Conversely, insufficient or biased sampling not only inflates representational error but also amplifies theoretical dispersion, undermining the reliability of any inferences drawn.

Thus, in Impossible Probability, the *control* of sample inversion (1/N or 1/∑xi) becomes a methodological necessity: it provides the statistical *safeguard* that ensures empirical ***investigation*** remains aligned with theoretical structure.

7.3. Infinite Universes and Limited Universes

The distinction between infinite universes and limited universes in *Impossible Probability* explains why the functions of sample inversion differ depending on the type of universe under study.

In **universes of infinite subjects or options**, the sample corresponds directly to N, the number of subjects or options included. Here, the inversion of N (1/N) performs two simultaneous functions under normal conditions:

1. It expresses the **probability of error in sample representativeness**, since larger N reduces error while smaller N increases it.
2. It expresses the **probability of theoretical dispersion**, since larger N reduces the divergence between empirical and theoretical distributions.

In **universes of limited options**, the situation changes fundamentally. Because N is predetermined by the model (e.g., two outcomes in a *coin* toss, six in a standard dice, a fixed number of candidates in an **election**), the true sample is not N but the total of direct scores or frequencies (∑xi). In this case, the inversion of frequencies, 1/∑xi, takes on the same dual role:

1. It represents the probability of error in sample representativeness.
2. It represents the probability of theoretical dispersion.

Despite these differences, a **universal rule remains valid** across all universes: the inversion of N (1/N) always retains its fundamental role as the **theoretical probability of occurrence by chance under conditions of equal opportunity**, and as the

arithmetic mean of empirical probabilities. Within the framework of the Second Method of Impossible Probability, empirical probability is defined as the ratio between the direct value or frequency of a given option and the total sum of direct values or frequencies:

$$p(x_i) = \frac{x_i}{\sum x_i}.$$

Thus, while the operational meaning of inversion varies depending on whether the universe is infinite or limited, its universal function—as a measure of equal theoretical probability and as the mean of empirical outcomes—remains inalienable.

7.4. Universality of the Inversion of the Sample

Because the definition of the sample differs depending on the type of universe—N in infinite universes, ∑xi in limited universes—it follows that the specific form of inversion also differs. Nevertheless, a universal principle governs both cases.

In all universes, whether infinite or limited, the **probability of error in sample representativeness** and the **probability of theoretical dispersion under normal conditions** are equal to the inversion of the sample. Depending on the context, this means:

- **1/N** in universes of infinite subjects or options.
- **1/∑xi** in universes of limited options.

This universality highlights the ***centrality of inversion*** as a unifying principle in *Impossible Probability*. While the ***specific mechanics*** ***adapt*** to the structural features of the universe, the fundamental logic—*that* dispersion and representativeness are inversely proportional to sample size—remains consistent.

7.5. Inverse Relationship Between Sample and Error

In all cases, both the **probability of error in sample representativeness** and the **probability of theoretical dispersion** are dependent variables of the sample size. As the sample grows, the probability of error decreases; as the sample shrinks, the probability of error increases.

This produces a strict **inverse proportionality**:

$$\text{Error or Dispersion} \propto \frac{1}{\text{Sample Size}}$$

This rule applies universally, regardless of whether the sample is expressed as N in infinite universes or as ∑xi in limited universes.

The implications of this principle are twofold:

1. **For infinite universes**, increasing N means greater representativeness and reduced dispersion, since larger subsets approximate the infinite whole.
2. **For limited universes**, increasing ∑xi (more observed occurrences) reduces error and dispersion, since the growing dataset stabilizes empirical distributions against theoretical expectations.

Thus, the inverse relationship between sample size and error constitutes one of the foundational laws of *Impossible Probability*. It establishes a clear methodological guideline: ensuring the sufficiency of the sample is the primary safeguard against error, regardless of the type of universe being analyzed.

7.6. Generic Relationship Between Sample and Error- Dispersion

In general terms, and regardless of the type of sample or the class of universe under consideration, the relationship between sample size and the probability of error in representativeness or theoretical dispersion is always **inversely proportional**.

This means that:

- As the sample size increases, the probability of error and theoretical dispersion decrease.
- As the sample size decreases, the probability of error and theoretical dispersion increase.

Thus, the control of error in representativeness and theoretical dispersion under normal conditions depends directly on ensuring that the sample is sufficiently large. The **rational sufficiency of the sample**—its capacity to be both statistically significant and structurally representative—is the methodological key to reducing both sources of error simultaneously.

This generic rule *holds across* both types of universes:

- In infinite universes, where the sample is N itself.
- In limited universes, where the sample is ∑xi, the sum of observed frequencies.

In both cases, the *principle* of inverse proportionality defines the logic of reliability.

7.7. First Method of Control: Sample Sufficiency

The **first method of controlling error** in sample representativeness and theoretical dispersion under normal conditions is through the **study of sample sufficiency**.

Sufficiency requires that the **magnitude** of the selected sample be large enough to minimize error, ensuring that the observed distribution reliably approximates the theoretical distribution. Without sufficiency, even the most carefully designed study risks generating distorted results.

At the same time, a second complementary mechanism of control exists: the requirement that sample selection be performed **randomly**. Randomness prevents structural bias from entering the sample and ensures that no option or subject is artificially overrepresented or underrepresented.

Thus, sufficiency and randomness *together* constitute the two foundational pillars of control in *Impossible Probability*.

7.8. Random Selection as a Method of Control

A sample that is either **insufficient in size** or **not randomly selected** will necessarily be biased. The origin of the bias lies in the method of selection itself:

- If the sample is too small, *it cannot capture the diversity* of the universe.
- If the sample is not random, the distribution of outcomes reflects the *preconceptions or constraints* of the selection method rather than the actual structure of the universe.

Therefore, randomness is not a secondary condition but an essential criterion. Without it, even a large sample can produce systematic distortions, undermining the reliability of the analysis.

7.9. Reliability of the Sample

A sample is **reliable** only if it meets two simultaneous conditions: it must be sufficiently large (sufficiency) and selected randomly (randomness).

If a sample is insufficient, or if it is not random—or worse, if it is both insufficient and non-random—then the bias observed in measurements will be a direct product of the sample selection process. In such cases, the study does not reflect the properties of the universe but rather the defects of the sampling method itself.

Reliability, then, is a dual requirement: a ***rationally sufficient sample*** that is also ***selected at random***. Without both conditions being met, representativeness is compromised, and statistical inference becomes invalid.

7.10. Insufficient Sample

An **insufficient sample** is defined as one that is not large enough to reduce the error of sample representativeness and theoretical dispersion under normal conditions.

In practical terms, insufficiency means that the sample does not adequately capture the variability of the universe. For example:

- In infinite universes, N is too small to provide representativeness.
- In limited universes, ∑xi is too small to stabilize empirical probabilities against their theoretical expectations.

The consequence of insufficiency is that dispersion remains artificially high, and the conclusions derived from the study are unreliable.

Thus, ensuring sufficiency is not only a technical requirement but a methodological ***safeguard***: without it, the very possibility of drawing generalizable conclusions is ***undermined***.

7.11. Non-Random Sample

A **non-random sample** arises when a study is not a study of an entire population—yet the sample is not chosen at random.

In such cases, the bias observed in the study can often be traced back not to the properties of the universe itself but to the **sampling technique** employed.

[A good example is marketing, if we want to sell a new product, first we need to select our target, so our study is focused on our target, from the beginning we are going to target that population with adds through AI to study the behaviour of the targeted population with our marketing strategy, from the very beginning the sample is biased and the target is to study the behaviour of our target in relation to our new product already on the market]

7.12. Insufficient and Non-Random Sample

A sample that is **both insufficient in size** and **non-random in selection** produces compounded bias. In this case, the distortions result simultaneously from:

1. The inadequacy of the sample size, which prevents the reduction of representativeness error.
2. The lack of randomness, which embeds systematic bias into the distribution of outcomes.

Such a sample cannot be considered representative of the universe it aims to represent. The conditions for representativeness are twofold: the sample must be rationally sufficient to reduce error, and it must be chosen randomly to eliminate bias. Without fulfilling both conditions, no inference drawn from the sample can be trusted.

Thus, the principle of *Impossible Probability* insists: **a sample only represents its universe if it is both sufficient and random**. Any failure to meet either condition undermines the legitimacy of statistical generalization.

[There may be situations in which, the sample is not large, just a few subjects or options, and not random, but we still need to carry out our research. In this situations we can say that under this conditions and context we observe a specific behaviour, obviously it is not a research to be applied to the whole universe, but only in those similar contexts were we can replicate the conditions to achieve same results. This scenario is more suitable for strict experimentation or quasi experimentation, where given a same context, laboratory or specific setting, we can replicate the same conditions to get the same results]

7.13. Requirements in Sample-Based Studies

With the exception of **population studies**—which examine entire universes and therefore do not require sampling—every study that depends on sampling must adhere to the strictest possible standards of sufficiency and randomness [as we said, specific laboratory experiments or quasi experiments are not going to meet the criteria, but as

long as we can apply the same conditions in similar contexts getting the same or similar results, we can describe these experiments in successful terms].

Only under these sampling conditions can the statistical decisions derived from ***rational critique of the sample be extrapolated and generalized*** to the broader universe that the sample is meant to represent.

If sufficiency or randomness is lacking, statistical inference is invalidated. Without representativeness, there is no rational bridge between the sample and the universe.

Therefore, in sample-based studies, the **necessary conditions** for valid inference are unequivocal: the sample must be **sufficient in size** and **random in selection**. These two criteria are inseparable, and together they form the methodological foundation for trustworthy statistical reasoning.

7.14. Rational Critique and Moral Representativeness

To determine whether a sample is sufficiently representative to reduce the error of sample representativeness to acceptable levels, it is necessary to apply a **rational critique** of the sample. Techniques of rational critique will be presented in subsequent sections, offering methodological criteria for evaluating sufficiency and randomness.

In any case, *Impossible Probability* emphasizes that every sample must not only be rationally sufficient but also **morally representative**. Moral representativeness requires that the sample be selected at random.

This *ethical* dimension acknowledges that statistical representativeness is *not purely a technical matter but also a matter of fairness*: only random selection ensures that no subject, option, or outcome is *unjustly privileged or excluded*. Thus, for a sample to be both *statistically valid and ethically defensible, it must be rational in its magnitude and random in its selection*.

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8. Level of Bias

8.1. Object of Study of Impossible Probability

The statistics of probability, or statistical probability, which in this framework is called Impossible Probability, takes as its object of study the *behavior* of phenomena understood through their probabilistic patterns. The quantitative manifestation of these behaviors appears in their direct values or frequencies, which constitute the positive form in which probabilistic reality reveals itself.

In other words, *Impossible Probability* does not study phenomena in their isolated or qualitative essence, but rather in the way they occur, repeat, and distribute themselves across time or populations. The frequency of occurrence becomes the first measurable sign of probabilistic reality. From this starting point, it is possible to build models that do not merely describe what happens, but also explain how probabilities emerge and evolve.

Thus, the object of study is not simply the occurrence of events, but their probabilistic *structure*: the way in which randomness, regularity, and deviation are interwoven in the observable data. This shift from isolated observation to probabilistic structure is one of the defining characteristics of Impossible Probability.

8.2. Empirical Probability and Relative Frequency

In Impossible Probability, empirical probability is defined as the ratio of the direct values or frequency of a particular subject or option to the total number of observations. This is the foundation of what is called the Second Method.

By contrast, traditional statistics—what here is referred to as the First Method—uses the concept of relative frequency in a much narrower sense. For classical statistics, relative frequency applies almost exclusively to the study of options in terms of proportions within a finite dataset.

Impossible Probability expands this notion into a universal principle: the ratio of the part to the whole is not restricted to frequency counts, but applies to every manifestation of probabilistic *behavior*. Whether we analyze direct values, aggregated outcomes, or distributions across categories, the same structural logic holds:

$$p(x_i) = \frac{x_i}{\sum x_i}$$

where p(xi) is the empirical probability of a given subject or option, xi its observed frequency, and ∑xi the total frequency of all subjects or options in the study.

By universalizing the part-to-whole relationship, Impossible Probability reinterprets empirical probability not as a local or descriptive measure, but as a general expression of how phenomena behave probabilistically. This universality is what allows the Second Method to compare across different types of universes—whether infinite in subjects or limited in options—using a common framework.

8.3. Probable Behavior and the Tendency Toward Normality

The probable behavior of what happens is typically equal to the direct value or frequency of a phenomenon divided by the total number of occurrences. This formulation highlights the structural *link* between *observed* frequency and *expected* probability.

A subject or option, under conditions of *equality* of opportunity, will tend toward normality as the sample size grows. In infinite universes, this implies that *N must tend*

toward expansion: the larger the N, the stronger the system's movement toward ***equilibrium*** and normal distribution.

This *principle* can be illustrated through several examples:

- Educational assessment: If a student achieves a certain *score* on an exam, their probable performance on another exam of similar characteristics—given *their level of knowledge*—will approximate the ratio of their score to the sum of scores obtained by all students in the same exam. Naturally, as the number of students in the sample increases, the overall pattern tends toward normality.
- Climatological observation: If, during a given year, a measurable quantity of rainfall is *recorded in the Sahara* Desert, the probable annual rainfall behavior can be defined as the ratio of rainfall that year to the total rainfall measured across the years included in the study. Once again, the larger the sample of years, the stronger the convergence toward a normal, stable pattern.

From these examples, the principle is clear: the greater the *size* of the *sample*, the greater the tendency toward normal probabilistic *behavior*. What begins as the randomness of individual events becomes structured, *stable*, and predictable when observed in sufficiently large aggregates.

In summary, Impossible Probability reframes empirical data as the foundation of a ***general law*** of tendency: under equal conditions, samples of increasing size produce convergence toward normality. This principle bridges the gap between empirical observation and theoretical expectation, forming the backbone of the concept of bias and its measurement.

8.4. Normality and Sample Sufficiency

A behavior is said to tend toward **normality** when the **empirical probability** of each subject or option converges toward the **theoretical inversion of N (1/N)**. Whether this convergence occurs depends on several factors, the most important of which is **sample sufficiency**.

- In universes of infinite subjects or options, sufficiency refers to the size of N itself: the greater the N, the more the empirical distribution tends toward the theoretical norm.
- In universes of limited options, sufficiency refers to the **total number of observed frequencies (∑xi)**. The larger the accumulated frequencies, the closer the system moves toward equilibrium.

Sample sufficiency therefore constitutes the threshold between randomness and structure: only when the sample is large enough can the probabilistic tendencies of the system be **meaningfully assessed**. Without sufficiency, neither equality of opportunity nor deviation from it can be measured reliably.

8.5. Example of the Dice and Normal Behavior

The classic case of a **six-sided dice** illustrates how sufficiency ensures normal probabilistic behavior. The universe here is limited to six possible options, one for each face of the dice.

If the dice is rolled only a few times, outcomes may appear irregular, with certain numbers occurring more frequently than others. However, as the number of rolls increases—approaching a sufficient total—the **empirical probability** of each face (1, 2, 3, 4, 5, or 6) converges toward the theoretical inversion of N, which in this case is 1/6.

Mathematically:

$$p(x_i) = \frac{x_i}{\sum x_i}$$

where p(xi) is the empirical probability of rolling face *i*, xi is the observed frequency of that face, and ∑xi is the total number of rolls.

The greater the number of rolls, the greater the probability that the distribution will approximate normality. This illustrates the fundamental law: **sufficiently large samples drive empirical outcomes toward their theoretical baseline**.

8.6. Definition of Empirical Probability

Empirical probability, in the framework of Impossible Probability, is defined as the **ratio of the direct value or frequency of a specific event to the total sum of direct values or frequencies of all events in the sample**.

This measure provides the simplest and most direct expression of how often a particular event occurs relative to the whole. It transforms raw counts into probabilistic structure, bridging the gap between observed data and theoretical expectation.

Thus:

$$p(x_i) = \frac{x_i}{\sum x_i}$$

captures not only the occurrence of events but also their relative weight in the system, serving as the foundation for evaluating normality, equality of opportunity, and bias.

8.7. Difference with Traditional Statistics

The principal difference between **Impossible Probability** and classical or traditional statistical models lies in their **object of analysis**.

- **Traditional statistics** (First Method) focuses primarily on the **behavior of direct values or frequencies**. Its central tools—means, variances, and other descriptive measures—are designed to analyze frequency distributions as they are [also, it is important to say that the First Method does not apply to the

mathematical analysis of raw data the probabilistic methods used for studies with limited options, normally].

- **Impossible Probability** (Second Method), by contrast, shifts the focus to the **behavior of empirical probabilities themselves**.

This shift has profound implications. Impossible Probability does not restrict itself to describing raw distributions but instead studies how probabilities behave relative to their theoretical expectations. This behavior may:

- Follow a **normal tendency toward equality of opportunity**, provided sufficient sample size and equal conditions.
- Exhibit **bias**, which can manifest as either positive or negative, depending on whether empirical probability exceeds or falls short of the theoretical inversion of N.

In this way, Impossible Probability integrates descriptive frequency analysis with normative probability structures, allowing for a richer and more universal interpretation of probabilistic phenomena.

8.8. Equality of Opportunity and Bias

A behavior is said to follow **equality of opportunity** when the **empirical probability** of each subject or option tends toward the theoretical inversion of N (1/N).

Conversely, a behavior is said to exhibit **bias** when empirical probability diverges from 1/N. Bias manifests in two distinct forms:

1. **Positive bias:** when the empirical probability of an option exceeds 1/N. This indicates that the option is overrepresented relative to equal opportunity.
2. **Negative bias :** when the empirical probability of an option falls below 1/N. This indicates underrepresentation relative to equal opportunity.

This distinction introduces a systematic way of diagnosing the probabilistic behavior of phenomena: whether they adhere to equality, deviate upward, or deviate downward. It

is the foundation for the formalization of the **Level of Bias**, which quantifies these deviations and transforms them into measurable variables for scientific analysis.

8.9. Exact Condition of Equality of Opportunity

A subject or option is said to behave **exactly as expected under equality of opportunity** when its **empirical probability** is equal to the **theoretical inversion of N (1/N)**.

$$p(x_i) = \frac{1}{N} \quad \rightarrow \quad \text{Statistical behavior under equality of opportunity}$$

This formulation defines the ideal condition of normality: every option in the universe has an equal share of probability, with no option overrepresented or underrepresented. Equality of opportunity thus constitutes the **baseline model** against which deviations are measured.

8.10. Definition of Positive Bias

A subject or option is said to display **positive bias** when its **empirical probability** exceeds the theoretical inversion of N.

$$p(x_i) > \frac{1}{N} \quad \rightarrow \quad \text{Positive bias}$$

This condition indicates that the subject or option occurs more frequently than would be expected under strict equality of opportunity. Positive bias represents

overrepresentation, suggesting either structural favoritism in the system or a probabilistic anomaly.

Mathematically, the measure of positive bias can be expressed as the difference between the empirical and theoretical probabilities:

$$\text{Positive Bias} = p(x_i) - \frac{1}{N} > 0$$

8.11. Definition of Negative Bias

Conversely, a subject or option is said to display **negative bias** when its **empirical probability** is less than the theoretical inversion of N.

$$p(x_i) < \frac{1}{N} \quad \rightarrow \quad \text{Negative bias}$$

This condition indicates that the subject or option occurs less frequently than would be expected under equality of opportunity. Negative bias represents **underrepresentation**, which may be due to structural disadvantage, exclusion, or natural asymmetry in the system.

Formally, negative bias is measured by the same differential:

$$\text{Negative Bias} = p(x_i) - \frac{1}{N} < 0$$

8.11.1. Differential Logic of Equality and Bias

The three conditions—equality, positive bias, and negative bias—can be unified under a single framework based on the difference between **empirical probability** and **theoretical probability**:

1. **Equality of opportunity:**

$$(p(x_i) = \tfrac{1}{N}) - \tfrac{1}{N} = 0$$

2. **Positive bias:**

$$(p(x_i) > \tfrac{1}{N}) - \tfrac{1}{N} = (+)$$

3. **Negative bias:**

$$(p(x_i) < \tfrac{1}{N}) - \tfrac{1}{N} = (-)$$

Thus, the **sign of the differential** (positive, zero, or negative) determines whether the probabilistic behavior corresponds to equality, overrepresentation, or underrepresentation.

This logic is foundational for the definition of the **Level of Bias** , which operationalizes bias into a measurable statistic:

$$\text{Level of Bias} = p(x_i) - \frac{1}{N}$$

8.12. Level of Bias as an Analytical Criterion

From the definitions established earlier, it follows that one of the most effective ways of determining whether a phenomenon in reality behaves under **equality of opportunity** or under **bias** (positive or negative) is by calculating its **Level of Bias**.

The Level of Bias is defined as the **difference between the empirical probability and the theoretical probability**, where the latter is expressed as the inversion of N:

$$\text{Level of Bias} = p(x_i) - \frac{1}{N}$$

This simple yet powerful formula provides a universal criterion for *classification*. Its interpretation is straightforward:

- If the **Level of Bias is zero**, the phenomenon follows a model of **equality of opportunity**.
- If the **Level of Bias is positive**, the phenomenon follows a model of **positive bias**, meaning that the option occurs more often than expected.
- If the **Level of Bias is negative**, the phenomenon follows a model of **negative bias**, meaning that the option occurs less often than expected.

The Level of Bias thus translates the abstract concept of deviation into a precise numerical value. It offers a methodological bridge between theory (1/N as the standard of equality) and empirical observation (p(xi)), providing a rigorous basis for analysis.

8.13. Typology of Tendencies According to the Level of Bias

Once the Level of Bias has been calculated, it becomes possible to determine the **directional tendency** of the phenomenon:

1. **Tendency toward equality of opportunity:** when the Level of Bias is zero, indicating that empirical probability aligns perfectly with the theoretical inversion of N.
2. **Tendency toward increasing positive bias:** when the Level of Bias is positive, showing that empirical probability is rising above the theoretical standard.
3. **Tendency toward increasing negative bias:** when the Level of Bias is negative, showing that empirical probability is descending further below the theoretical standard.

This typology transforms the Level of Bias from a static measure into a **dynamic diagnostic tool**. It not only identifies whether bias exists but also indicates its direction and intensity.

In practice, this allows researchers to *classify the behavior* of any subject, option, or phenomenon according to whether it tends toward equilibrium, overrepresentation, or underrepresentation. Such classification is indispensable in fields as diverse as *demography, political science, psychology, or machine learning*, where deviations from equal distribution often carry *structural, ethical, or functional significance*.

8.14. Inversion of N as a Criterion of Tendency

The inversion of N (1/N) thus emerges as the **fundamental benchmark** for evaluating the tendencies of phenomena. By serving as the universal standard of equality of opportunity, it allows us to determine whether a subject or option behaves:

- **According to the ideal of equality of opportunity**, when the Level of Bias is zero.
- **Above the norm**, when the Level of Bias is positive, indicating overrepresentation.
- **Below the norm**, when the Level of Bias is negative, indicating underrepresentation.

The degree or intensity of equality or bias is measured by the **differential between empirical and theoretical probability**. In this way, inversion of N provides not only a baseline of normality but also a scale for quantifying departures from it.

This approach establishes a unified statistical framework in which the **presence, absence, and direction of bias** can be evaluated with precision. By anchoring analysis in 1/N, Impossible Probability transforms the abstract ideal of equal opportunity into a concrete criterion for identifying and measuring probabilistic tendencies in real-world data.

8.15. Difference Between Differential Score and Level of Bias

In traditional statistics—the **First Method**—the **differential score** is defined as the difference between the **direct score (or frequency)** of a subject or option and the **arithmetic mean** of all scores. Symbolically:

$$\text{Differential Score} = x_i - \bar{x}$$

This measure evaluates how much an individual observation deviates from the average of the entire sample. It is therefore a relative measure, grounded in the relationship between each score and the central tendency of the dataset.

By contrast, in **Impossible Probability (the Second Method)**, the focus shifts away from direct scores and their arithmetic mean. Instead, the relationship is established between **empirical probability** and **theoretical probability** under equality of opportunity. The corresponding measure is the **Level of Bias**, defined as:

$$\text{Level of Bias} = p(x_i) - \frac{1}{N}$$

This reformulation is crucial: while the First Method interprets deviation with respect to an internal property of the dataset (its mean), the Second Method evaluates deviation with respect to an **external normative criterion**: the inversion of N, which embodies the principle of equality of opportunity.

Thus, the **differential score** measures variability within the dataset, whereas the **Level of Bias** measures deviation from the theoretical standard of fairness.

8.16. Difference in Zero-Sample Cases Between the First and Second Method

A particularly illuminating difference between the two methods arises in the case of **zero-samples**, where all direct scores or frequencies are equal to zero.

- **In the First Method (traditional statistics):**
  If all subjects or options in the sample have a direct score of zero, the arithmetic mean will also be zero. Consequently, every differential score will be:

$$(x_i = 0) - \bar{x} = 0$$

This implies that all deviations are zero, and measures of variability such as the mean deviation or standard deviation are also zero. In other words, the First Method interprets the zero-sample as perfectly uniform and devoid of dispersion.

- **In the Second Method (Impossible Probability):** The situation is interpreted very differently. If all subjects or options have a direct score of zero, then the total sum of scores (∑xi\sum x_i∑xi) is also zero. This means that **empirical probability** is zero for all options:

$$p(x_i) = \frac{0}{\sum x_i} = 0$$

However, the **Level of Bias** is not zero, because the **theoretical probability** under equality of opportunity (1/N) remains greater than zero. Thus:

$$\text{Level of Bias} = p(x_i) - \frac{1}{N} = 0 - \frac{1}{N} = -\frac{1}{N}$$

In this case, every option is maximally underrepresented relative to the theoretical baseline. The result is what can be called the **Maximum Possible Negative Bias**:

$$\text{Maximum Negative Bias} = -\frac{1}{N}$$

Since all options share this negative bias equally, the **average absolute Level of Bias** (equivalent to the mean deviation in Impossible Probability) is:

$$\text{Mean Deviation of Bias} = \frac{\sum |p(x_i) - \frac{1}{N}|}{N} = \frac{1}{N}$$

This example demonstrates the **fundamental distinction** between the two methods. Traditional statistics interprets the zero-sample as a case of *perfect equality with zero deviation*, while Impossible Probability interprets it as a case of *maximum negative inequality*, since all options fall short of their theoretical probability by exactly 1/N.

The difference lies in the reference point:

- The First Method compares data to the internal mean, which in this case is also zero.
- The Second Method compares data to the external theoretical norm of equality of opportunity, 1/N, which remains positive even when all empirical probabilities are zero.

Thus, Impossible Probability reveals a **hidden asymmetry** in cases where traditional statistics sees none, highlighting its capacity to detect bias not only within distributions but also with respect to normative probabilistic ideals.

8.17. Maximum Possible Negative Bias and Mean Deviation

The case of a **zero-sample** (where all empirical probabilities equal zero) produces what can be termed the **Maximum Possible Negative Bias**. This occurs because the empirical probability of each option is zero, while the theoretical probability under equality of opportunity remains 1/N.

Formally:

$$\text{Maximum Negative Bias} = \left| \left(p(x_i) = 0\right) - \frac{1}{N} \right| = \frac{1}{N}$$

This measure defines the absolute limit of underrepresentation for any subject or option. It sets a lower bound on the scale of bias, just as positive overrepresentation defines the upper bound.

Furthermore, when every option in the sample shares this condition equally, the **average Level of Bias** (equivalent to the mean deviation in Impossible Probability) is also equal to 1/N. This is because the uniformity of the bias means that the deviation is constant across all options.

Thus, Maximum Negative Bias provides both a diagnostic tool and a theoretical anchor: it identifies the ***extreme scenario in which no option achieves its theoretical probability***, and all fall short by exactly the same amount.

8.18. Applied Example: A Sample of Patients Without Symptoms

To illustrate, consider a medical study in which a new experimental treatment produces the outcome that **all patients display zero symptoms**.

From the perspective of traditional statistics, such a dataset would be described as perfectly uniform: every score is zero, the mean is zero, and all deviations are zero. Variability is absent, and the sample would be seen as fully homogeneous.

From the perspective of Impossible Probability, however, the analysis yields a different conclusion. The **empirical probability** of each symptom is zero, but the **theoretical probability** (1/N) remains positive. Therefore, each option (possible symptom) exhibits the **Maximum Negative Bias**:

$$\text{Level of Bias} = (p(x_i) = 0) - \frac{1}{N} = -\frac{1}{N}$$

Since this condition applies to every option equally, the **mean deviation of the Level of Bias** is:

$$\text{Mean Deviation} = \frac{\sum \left| (p(x_i) = 0) - \frac{1}{N} \right|}{N} = \frac{1}{N}$$

Thus, while classical statistics interprets the sample as a case of zero deviation, Impossible Probability interprets it as a case of **maximum underrepresentation** relative to theoretical expectations.

This example underscores how Impossible Probability can reveal hidden structures of bias in data that traditional approaches treat as neutral or trivial.

8.19. Studies of Probability: Equality of Opportunity or Bias

In Impossible Probability, every study is, at its core, a **study of probabilities**. With the exception of special cases that adapt the framework to direct scores or frequencies (see Chapter 12 [*Introduction to Impossible Probability*]), the focus is always on evaluating whether a phenomenon follows a tendency toward:

1. **Equality of opportunity:** when empirical probabilities converge toward 1/N, producing minimal or zero bias.
2. **Positive or negative bias:** when empirical probabilities diverge from 1/N, producing overrepresentation or underrepresentation.

Within studies of bias, two major categories can be distinguished:

- **Normal studies:** where dispersion oscillates between two poles:
    - **Zero dispersion**, when probabilities tend to equality of opportunity.
    - **Maximum dispersion**, when probabilities diverge to their theoretical limits.
- **Omega models:** where dispersion does not follow the normal oscillatory pattern and may incorporate additional structural dynamics [ideals between 2 and N-1]. These will be elaborated in later chapters.

In this way, Impossible Probability systematizes all probabilistic studies as inquiries into the **tension between equality and bias**. This duality—equality as the theoretical baseline and bias as deviation—provides a unified framework for analyzing randomness, structure, and deviation in both empirical and theoretical domains.

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9. Maximum Theoretical Mean Deviation Possible and Maximum Theoretical Standard Deviation Possible

9.1. The Logic of the Maximum Theoretical Mean Deviation Possible

The concept of the Maximum Theoretical Mean Deviation Possible holds a pivotal role in the framework of *Introduction to Impossible Probability*. It is systematically addressed in Chapters 10 and 11 of the work, where the logic of dispersion under extreme conditions is fully elaborated. From this foundation, Chapter 11 proceeds to derive a closely related concept: the Maximum Theoretical Standard Deviation Possible.

The reasoning behind this notion is straightforward yet profound: if dispersion is understood as the average level of bias or deviation within a distribution, then there must exist an upper boundary to which such dispersion can tend under conditions of maximum inequality. The maximum theoretical mean deviation provides precisely this benchmark. It identifies the mathematical limit that dispersion can reach when the normative ideal of the scientific policy guiding the study is not equality of opportunity but, on the contrary, the concentration of probability into a single option or subject at the expense of all others.

By formalizing this extreme case, Impossible Probability establishes a reference point that allows researchers to determine where real empirical data stand: whether they fall closer to conditions of equality (minimum deviation) or to conditions of maximum concentration (maximum deviation). This logic provides the groundwork for the subsequent formulation of the maximum theoretical standard deviation, which refines

the analysis by considering squared deviations and their square root. Together, these two measures define the limits of possible dispersion within the model.

9.2. Error Margins in Impossible Probability

Impossible Probability introduces itself as an *alternative model* of statistical reasoning that studies probability not only as a mathematical abstraction but as a means of explaining what actually happens under conditions of uncertainty. In doing so, it explicitly acknowledges the presence of error margins, treating them not as accidental imperfections but as intrinsic components of probabilistic knowledge.

Three kinds of error margins are central to this framework:

1. Sampling error in universes of infinite subjects or options: measured by the inversion of N, that is, 1/N, where N corresponds to the size of the sample.
2. Sampling error in universes of limited options: measured by the inversion of the total sum of direct scores or frequencies, $1/\sum x_i$, where $\sum x_i$ represents the observed sample size in terms of actual occurrences.
3. Rational margin of error or critical ratio: an additional measure established by scientific policy, which determines the degree of rational doubt that can be tolerated in order for a conclusion to be accepted as valid.

The incorporation of these margins of error serves a crucial purpose: it defines the conditions under which knowledge can be considered objective. A sample that is sufficiently large and rational does not merely describe local phenomena—it acquires universal validity, enabling the construction of causal models that *explain the general behavior of systems* on the basis of particular, representative cases.

Thus, Impossible Probability combines two elements often treated separately: the empirical uncertainty inherent to sampling and the rational requirement of universality in science. The result is a statistical framework that is at once pragmatic and philosophical, rooted in error margins yet oriented toward the pursuit of objective, causal explanation.

9.3. Behavior as the Object of Study

At the core of Impossible Probability lies the concept of ***behavior*** (***comportamiento***) [this is foundational in our psychological model: *condicion, context. Comportamiento*], understood as the ***fundamental*** object of study. Everything that exists reveals itself by "being what it is," and this being can be measured quantitatively through its direct value or frequency. By dividing the direct value or frequency of a particular subject or option by the total direct values or number of occurrences, one obtains its empirical probability, which reflects the probable behavior of that phenomenon within a given sample.

When the sample is sufficiently large and rationally significant, empirical probability becomes more than a descriptive statistic—it constitutes a rational reason for truth. Yet, this truth is not absolute. It exists within the boundaries of rational doubt or empirical skepticism, which *scientific policy must define and accept* as part of its methodological stance.

This *epistemological structure implies an ethical dimension*: every acceptance of empirical truth entails a political and moral *responsibility*. Scientific conclusions are never purely technical—they reflect choices about how much uncertainty to tolerate and about what counts as sufficiently rational evidence. Thus, Impossible Probability positions itself as more than a statistical method: it is a **framework that ties empirical measurement to philosophical reflection and political responsibility**.

In this way, *behavior* is not only *quantified* but also *contextualized* within a system of rational acceptance, skepticism, and responsibility, making the Probabilidad Imposible a uniquely interdisciplinary model that ***unites mathematics, epistemology, and ethics***.

9.4. Behavior and Tendency in Impossible Probability

Behavior, in the framework of Impossible Probability, is not merely a static measurement but manifests itself dynamically as **tendency**. Every system, subject, or

option has a way of behaving that can be described probabilistically, and this *behavior* inevitably reveals a *direction or inclination over time*.

In Impossible Probability, the **tendency of behavior** is determined by the **ideals** that guide the scientific model under study. If these ideals conform to what is defined as a **normal model of dispersion**, then dispersion will oscillate between two polar conditions:

- **Zero dispersion**, when the ideal is equality of opportunity, meaning all subjects or options have equal empirical probability, converging on 1/N.
- **Maximum dispersion**, when the ideal is the concentration of probability in a single subject or option defined a priori as the ideal, while all others tend to zero.

Exceptions to these patterns include:

1. **Zero-sample models**, where all subjects or options ideally tend to empirical probability zero (a special case treated separately) [for instance, zero symptoms when treating medical *conditions*].
2. **Omega models (Ω)**, *where* dispersion does not conform to the normal range between zero and maximum but instead follows a distinctive distribution determined by a subset of multiple ideals within the universe.

Thus, *behavior* in Impossible Probability is always studied in relation to these structural ideals, which define the expected trajectory of empirical outcomes.

9.5. Conditions of Maximum Dispersion

The present section, *centered* on the **Maximum Theoretical Mean Deviation Possible**, seeks to clarify under what conditions such maximum deviation may arise. These are referred to as **conditions of maximum dispersion**, which are characterized as follows:

Maximum dispersion occurs when the scientific ideal that frames the study explicitly directs the system toward the greatest possible inequality in the distribution of probabilities. In other words, the system is structured so that one option absorbs the maximum possible empirical probability, while all others collapse to zero.

Studying these conditions allows us to determine the absolute theoretical limits of dispersion. This serves as a reference point for empirical studies, making it possible to measure whether real-world samples approach or diverge from such extreme configurations.

9.6. Formal Definition of Maximum Dispersion

Formally, **conditions of maximum dispersion** are said to exist when every subject or option within a normal model achieves the maximum possible bias assigned to it by the guiding ideals of scientific policy.

If we consider a sample of **N subjects or options**, maximum dispersion occurs when only **one single subject or option** attains the **Maximum Empirical Probability Possible**, which is equal to 1. Since no probability can exceed the value of unity, this outcome necessarily implies that all other subjects or options within N must hold **empirical probability equal to zero**, which corresponds to the **Minimum Empirical Probability Possible**.

$$p(x_i) = 1 \quad \rightarrow \quad \text{Maximum Empirical Probability Possible}$$
$$p(x_i) = 0 \quad \rightarrow \quad \text{Minimum Empirical Probability Possible}$$

This binary opposition (one option at probability 1, all others at 0) is the formal hallmark of maximum dispersion, representing the extreme case of inequality in probabilistic behavior.

9.7. Applied Example: Survey on Recycling

To illustrate this concept, consider a **survey on ecological attitudes**, where one particular item asks respondents: *Do you recycle?* The survey provides three options:

1. *Recycle*
2. *Do not recycle*
3. *Do not know or no answer*

If the **scientific ideal** underlying the study is to maximize the concentration of responses in the ecologically favorable option (*recycle*), then the model would predict a tendency toward **Maximum Empirical Probability Possible** for that option ($p = 1$), while the probabilities of the other two options would tend toward zero.

In such a scenario, as more individuals in the survey population adopt the ideal behavior (*recycle*), the sample would exhibit a growing degree of **empirical dispersion**, approaching the theoretical limit of **Maximum Empirical Dispersion Possible**.

This example demonstrates how the choice of **policy ideals** directly influences the probabilistic structure of a study: the closer empirical reality aligns with the designated ideal option, the more the sample's dispersion approximates its theoretical maximum.

9.8. Alternative Models of Maximum Dispersion

It is important to note that conditions of **maximum dispersion** are not limited to simple dichotomous or trichotomous models such as *recycle / do not recycle / no answer*. Other, more complex distributions can also be structured in such a way that the **ideal** remains the concentration of probability in a single category, thereby driving the system toward maximum dispersion.

For example, suppose instead of the three basic options, the categories in a survey are expanded into a more detailed scale:

- *Recycle always, in any circumstance*
- *Recycle only household waste*
- *Recycle occasionally*
- *Recycle rarely*
- *Never recycle*

In such a case, the distribution of categories allows for a finer gradation of behaviors, but the underlying principle of maximum dispersion remains the same: if the scientific ideal is that the population should overwhelmingly converge on one category (e.g., *Recycle always*), then the system tends toward maximum empirical dispersion, with the chosen option approaching probability 1 and all others approaching probability 0.

While such situations can still be analyzed through the **Second Method of Impossible Probability** (focused on empirical probabilities derived from frequencies), they may also lend themselves to alternative methods such as the **Effective Distribution** approach (explored in Section 22 of *Introduction to Impossible Probability*). This method enables more nuanced treatment of categorical gradations when the scientific policy requires it.

9.9. Omega Models and Subsets of Ideals

A second class of models departs from the **normal distribution of ideals** and requires a different analytical treatment. These are known as **omega models (Ω)**, discussed in Section 11 of *Introduction to Impossible Probability*.

Omega models occur when, within a universe of N subjects or options, there exists not one single ideal, nor the totality of N as ideals, but instead a **subset of ideals** greater than one and less than N. This subset is defined as the **omega subset (Ω)**.

For example, in a study of ecological attitudes with the following options:

- *Recycle*
- *Do not recycle*
- *Use no environmentally harmful chemicals*
- *Use harmful chemicals*
- *Respect the environment*
- *Do not respect the environment*
- *Enjoy nature*
- *Do not enjoy nature*
- *Do not know / no answer*

The subset of ecological ideals (Ω) could include: *Recycle, Use no harmful chemicals, Respect the environment, Enjoy nature*. The non-ecological attitudes, in contrast, would be: *Do not recycle, Use harmful chemicals, Do not respect the environment, Do not enjoy nature, No answer*.

In such a case, because the number of ideals is greater than one but less than N, the system does not tend toward maximum dispersion. Instead, it tends toward an **ideal omega dispersion**, where probability is distributed among a plurality of ideal categories. The omega framework thus expands the analytical toolkit of Impossible Probability, enabling the study of systems where several "positive ideals" coexist within the same probabilistic universe.

9.10. The Zero-Sample Phenomenon

A particularly distinctive phenomenon within Impossible Probability is the case of the **zero-sample**. This arises when the scientific ideal of a study is that all subjects or options should tend toward **empirical probability zero**.

Consider, for instance, a **medical trial of an experimental drug** where the goal is that all patients should ultimately exhibit zero symptoms. As the treatment progresses, most patients approach the ideal (zero frequency of symptoms). However, if even a minimal remainder of patients continues to exhibit non-zero symptoms—whether due to uncontrolled interactions, unique immune responses, or other interfering variables—then the empirical probability of these residual cases tends to rise relative to the rest of the distribution. This increase, in turn, drives **empirical dispersion upward**.

The result is that models where the ideal is a zero-sample behave in a **special and paradoxical way**:

- As frequencies decrease toward zero but do not reach absolute zero, dispersion actually tends toward its maximum.
- Dispersion only balances back to the theoretical inversion of N (1/N) at the exact point where **every subject or option without exception reaches empirical probability zero**.

Because of this anomaly, **zero-sample ideals are excluded from the class of normal models** of dispersion within Impossible Probability. They require special analytical treatment distinct from both normal models of equality/maximum dispersion and from omega models.

9.11. Definition of Normal Dispersion Models

Within the framework of Impossible Probability, **normal dispersion models** are defined as those in which dispersion tends either toward **zero [equality]** or toward its **maximum**. This definition deliberately **excludes two special cases**:

1. **Zero-sample models**, in which all probabilities tend to zero, producing paradoxical behavior (as explained in Section 9.10).
2. **Omega models**, in which dispersion is distributed across a subset of multiple ideals rather than concentrated in one option or spread equally across all.

By setting aside these exceptional cases, Impossible Probability establishes a **general rule**: in most empirical studies, dispersion will oscillate between the two poles—**perfect equality (zero dispersion)** and **maximum inequality (maximum dispersion)**. These constitute the **normal range of probabilistic behavior** in statistical analysis.

9.12. Political Ideals and Sample Dispersion

Dispersion, as it is conceptualized in Impossible Probability, is not only a statistical property but also the reflection of **scientific and political ideals**. The **ideal adopted by scientific policy** determines whether a sample tends toward equality or concentration:

- If the **ideal is equality of opportunity**, then the sample will tend toward **zero dispersion**, with each subject or option converging on the theoretical probability 1/N.
- If the **ideal is the concentration of probability in one subject or option**, designated in advance as the preferred or "ideal" option, then the system will tend toward **maximum dispersion**.

Thus, dispersion cannot be analyzed in isolation from the **philosophical and normative framework** of the study. The **political choice of ideals**—whether equality or concentration—defines the probabilistic structure of the sample.

Formally:

- Under the ideal of equality, the **Mean or Standard Deviation** will converge to **zero**.
- Under the ideal of concentration, the **Mean or Standard Deviation** will converge to their **maximum possible values**.

This highlights the epistemological claim of Impossible Probability: statistical measures are always tied to **value-laden decisions about what constitutes the "ideal" outcome**.

9.13. Standardization of Mean and Standard Deviation

Impossible Probability introduces a process of **standardization** for the concepts of **Mean Deviation** and **Standard Deviation**. This standardization reflects a shift in emphasis compared to classical statistics.

- In Impossible Probability, **Mean Deviation** is identified with the **Average Level of Bias**. It is calculated as the average of the sum of the **absolute values of all individual Levels of Bias**. This total sum of absolute biases is referred to as the **Total Bias**.
- In traditional or conventional statistics, by contrast, the emphasis has typically been placed on the **Standard Deviation**, which squares deviations, averages them, and then takes the square root.

By redefining Mean Deviation in terms of Levels of Bias, Impossible Probability integrates **empirical measurement** and **theoretical comparison** into a unified framework. This allows the measure of dispersion to directly reflect the degree to which empirical probabilities align (or fail to align) with the theoretical standard of 1/N.

9.14. Objectivity of Mean Deviation in the Second Method

From the perspective of Impossible Probability, **Mean Deviation** is considered more **objective** than Standard Deviation. The reason lies in its methodological construction:

- In the **Second Method** (Impossible Probability), Levels of Bias are not squared, nor is the calculation reduced to the square root of averaged squared differentials.
- Instead, deviations are measured in their **absolute values**, preserving their real magnitude without artificially inflating or reducing them.

This ensures that Mean Deviation represents a **faithful and transparent measure** of the true average difference within the sample. Unlike Standard Deviation, which transforms deviations and thus may obscure their immediate interpretive meaning, Mean Deviation in Impossible Probability remains a **direct expression of average bias**.

Moreover, each **individual bias** (the deviation of a subject or option from the theoretical value 1/N) represents the way in which that entity manifests its **individual tendency** within the sample. By aggregating these without distortion, Mean Deviation provides a clearer window into the **collective behavior of the system**.

Thus, Impossible Probability elevates Mean Deviation as the preferred measure of dispersion, positioning it as more consistent with the epistemological aim of capturing reality **as it is**, rather than through mathematical transformations

9.15. Tendency Toward Maximum Dispersion in Normal Models

In the framework of Impossible Probability, when a **normal model** is structured such that the ideal tends toward the **Maximum Empirical Probability Possible**, both the **Mean Deviation** and the **Standard Deviation** will necessarily move toward their **maximum possible values**.

This reflects a logical consequence of the model's construction: if the system is designed so that one subject or option absorbs the full concentration of probability, the statistical measures of dispersion must reflect the widest possible divergence between the theoretical expectation of equality (1/N) and the actual empirical outcome ($p(x) = 1$ for one option, and 0 for all others).

The **upper bound** of this process is not left undefined. Instead, it is formalized through two measures:

- The **Maximum Theoretical Mean Deviation Possible**, which captures the maximum average level of bias across the entire sample.
- The **Maximum Theoretical Standard Deviation Possible**, which expresses the same extremity of dispersion using the conventional measure of squared differentials and their root.

These constructs provide a **mathematical ceiling** for dispersion in normal models, allowing researchers to determine when an observed distribution approaches or achieves the theoretical extreme.

9.16. Definition of the Maximum Theoretical Positive Bias

Within a **normal bias model**, the **Maximum Theoretical Positive Bias** (*Máximo Sesgo Teórico Posible*) represents the largest deviation above equality that an **ideal subject or option** can achieve.

This condition arises when the empirical probability of the ideal subject or option reaches its **absolute maximum**:

P

$$p(x_i) = 1 \quad \rightarrow \quad \text{Maximum Empirical Probability Possible}$$

Since the theoretical benchmark under equality of opportunity is 1/N, the **Maximum Theoretical Positive Bias** is calculated as:

$$1 - \frac{1}{N} = \text{Maximum Theoretical Positive Bias}$$

This formula captures the essential logic of Impossible Probability:

- The **unit (1)** represents the certainty of the ideal option dominating completely.
- The subtraction of **1/N** reflects the difference between this outcome and the equal-probability baseline.

Thus, the maximum positive bias defines the greatest possible distance between the **empirical reality of concentration** and the **theoretical ideal of equality**.

9.17. Definition of the Maximum Theoretical Negative Bias

Conversely, the **Maximum Theoretical Negative Bias** (*Máximo Sesgo Negativo Posible*) arises when an option's **empirical probability equals zero**—that is, when the option becomes completely absent from the observed distribution:

$$p(x_i) = 0 \quad \rightarrow \quad \text{Minimum Empirical Probability Possible}$$

Under these conditions, the **Level of Bias** for that option equals the difference between zero and the theoretical baseline of equality, is −1/N. Expressed in absolute terms, this yields:

$$|0 - \tfrac{1}{N}| = \tfrac{1}{N} = \text{Maximum Theoretical Negative Bias}$$

This condition applies systematically to all **N – 1 non-ideal subjects or options** when one option is defined in advance as the sole ideal and absorbs all probability. Thus, in a maximally dispersed model, one option will exhibit the **Maximum Theoretical Positive Bias**, while all remaining options will simultaneously exhibit the **Maximum Theoretical Negative Bias**.

This dual structure—the coexistence of **one maximum positive bias** with **N – 1 maximum negative biases**—is central to the analysis of maximum dispersion in Impossible Probability. It establishes the precise statistical geometry of extreme inequality in probabilistic systems.

9.18. Formula for Mean Deviation or Average Level of Bias

In the **Second Method of Impossible Probability**, the **Mean Deviation** (Desviación Media) is defined as the **average of the Total Bias**, where **Total Bias** is the sum of the absolute values of all individual Levels of Bias.

If we imagine a **hypothetical case** where, in a population of size **N**, a single subject or option designated a priori as "ideal" reaches the **Maximum Empirical Probability Possible** (p(x) = 1), while all other **N – 1 subjects or options** attain the **Minimum Empirical Probability Possible** (p(x) = 0), then the system is at its most extreme configuration of inequality.

Under such conditions, and assuming that the **political ideal** of maximum dispersion is fully realized, the model must necessarily converge toward the **Maximum Theoretical Mean Deviation Possible** or the **Maximum Theoretical Standard Deviation Possible**.

Formally:

$$\text{Mean Deviation or Average Level of Bias} = \frac{\sum |p(x_i) - \frac{1}{N}|}{N}$$

$$\text{Total Bias} = \sum |p(x_i) - \frac{1}{N}|$$

This formula captures the **core principle** of Impossible Probability: dispersion is the **direct consequence** of the difference between the empirical distribution and the theoretical baseline of equality (1/N).

9.19. Calculation of the Maximum Theoretical Mean Deviation Possible

When the **political ideal of maximum inequality** is achieved—one subject or option reaches the maximum probability (1), while all others fall to zero—the Total Bias is maximized.

In this extreme configuration:

- The single "ideal" subject or option exhibits the **Maximum Theoretical Positive Bias** (1−1/N).
- The remaining N−1 non-ideal subjects or options each exhibit the **Maximum Theoretical Negative Bias** (1/N).

The **Maximum Theoretical Mean Deviation Possible** is then obtained as the **average** of these values:

$$\text{Maximum Theoretical Mean Deviation Possible} = \frac{(1 - 1/N) + [(1/N) \cdot (N - 1)]}{N}$$

This formula integrates both the **concentration of probability in the ideal subject or option** and the **absence of probability in all others**, averaged across the N elements of the system.

9.20. Relationship with Maximum Variance and Standard Deviation

The same logic applies when shifting from Mean Deviation to **Variance** and **Standard Deviation**.

- The **Maximum Theoretical Variance Possible** is derived by squaring each of the bias terms before summation. Specifically:
    - The Maximum Theoretical Positive Bias (1−1/N) is squared.

    - The Maximum Theoretical Negative Bias (1/N) is squared, then multiplied by N−1

- The **Maximum Theoretical Standard Deviation Possible** is the **square root** of this variance.

Thus, Impossible Probability establishes a **hierarchy of dispersion measures**—from Mean Deviation (linear in bias) to Variance and Standard Deviation (quadratic and square-root adjusted). This hierarchy allows the **contrast of hypotheses** based on the **Critical Ratio** (*Razón Crítica*), a concept developed in Section 11 of *Introducción a la Probabilidad Imposible*.

For example:

- In a **bias study**, one might test whether the observed dispersion grows sufficiently toward the maximum ideal by verifying if it equals or surpasses a Critical Ratio.
- In a **normal equality study**, the test would instead check whether dispersion tends sufficiently toward zero.

In both cases, the **Critical Ratio** is computed as the product of a chosen reliability percentage (X/100) and the maximum theoretical dispersion (mean or typical).

9.21. Simplification of the Maximum Theoretical Mean Deviation Equation

The formula for the **Maximum Theoretical Mean Deviation Possible** can be **simplified** by recognizing an underlying equivalence. The product of the **inverse of N** (1/N) by the differential **(N – 1)** is equal to the **Maximum Theoretical Positive Bias**.

This reflects a **principle of natural symmetry**: every negative event is counterbalanced by a positive one. In probabilistic terms, this means that **positive and negative biases compensate one another**, such that their algebraic sum would be zero. For this reason, Impossible Probability always calculates Total Bias using **absolute values** (or squared values in the case of variance).

$$\text{Maximum Theoretical Mean Deviation Possible} = \frac{2 \cdot (1 - 1/N)}{N}$$

This simplified expression emphasizes that the maximum average dispersion is essentially the **double of the Maximum Theoretical Positive Bias**, normalized across the population size N.

9.22. Alternative Expressions of the Maximum Theoretical Mean Deviation Possible

Another equally valid way of formulating the **Maximum Theoretical Mean Deviation Possible** is to express it directly as the **average of the double of the Maximum Theoretical Positive Bias**.

Formally:

$$\text{Maximum Theoretical Mean Deviation Possible} = \frac{[(1 - 1/N) \cdot 2]}{N}$$

This expression is equivalent to the more expanded version presented earlier:

$$\text{Maximum Theoretical Mean Deviation Possible} = \frac{(1 - 1/N) + [(1/N) \cdot (N - 1)]}{N}$$

Since algebraically, these two formulations reduce to the same result, the second is often preferred for **clarity of interpretation**, while the first is used for **mathematical simplicity**.

Another alternative is to express the formula in terms of the **inverse of N** multiplied by the differential N−1, again doubled and then averaged:

$$\text{Maximum Theoretical Mean Deviation Possible} = \frac{[(1/N)\cdot(N-1)\cdot 2]}{N}$$

This version emphasizes that the measure can be constructed from the **negative bias component** (the absence of probability in the N−1 non-ideal cases) rather than from the **positive bias component** (the concentration of probability in the single ideal case). In both interpretations, the outcome is the same, but each highlights a different **probabilistic mechanism**:

- Concentration (positive bias) in the ideal option.
- Depletion (negative bias) in all remaining options.

In summary, the three formulations can be considered **algebraically interchangeable**:

$$\frac{(1-1/N)+[(1/N)\cdot(N-1)]}{N} = \frac{[(1-1/N)\cdot 2]}{N} = \frac{[(1/N)\cdot(N-1)\cdot 2]}{N}$$

However, in the context of *Impossible Probability*, where the aim is always to **maximize the transparency of the theoretical structure**, the **most common form** of the equation is the one based on the **average of the double of the Maximum Theoretical Positive Bias**:

$$\text{Maximum Theoretical Mean Deviation Possible} = \frac{2 \cdot (1 - 1/N)}{N}$$

This preference reflects the broader methodological principle of the Second Method: mathematical formulations should remain as **clear and simplified as possible**, while retaining the capacity to highlight the **dialectical relationship** between equality of opportunities (1/N) and extreme deviation from that equality (maximum bias).

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10. Ideal Mean Deviation and Ideal Standard Deviation [Dispersion in Omega Models]

10.1. Science, Politics, and the Ideological Definition of Reality

Every scientific discourse is, in essence, a political discourse. In other words, every scientific theory is political because all science depends on a particular *scientific policy*. This policy differentiates one scientific model from another by providing an ideological definition of reality. Reality in itself is unknowable—except as mediated through human ***physiology. Reason*** admits only phenomena to be knowable, not the moral universe as it exists in itself. What science can do is construct, through rational critique, a universe of moral propositions about what "objectivity" is, although such objectivity can never be completely pure.

This framing emphasizes that scientific knowledge is not neutral. It is inseparably tied to the epistemic and ideological commitments of the political systems in which it is embedded. The very decision of how to define "objectivity" is not a purely descriptive act, but also a normative and political one. Thus, even within the statistical framework of *Impossible Probability*, which strives for rational precision, the limits of science are defined by the political choices that structure its assumptions.

10.2. Scientific Truth and Moral Propositions

Every scientific definition that claims to be truth is, ultimately, a moral proposition. To affirm something false is considered immoral; however, the "scientific morality" of any age does not claim absolute truths but only partially true propositions, always conditioned by a *margin of error* inherent to hypothesis testing.

The advancement of science lies in progressively narrowing this margin of error within the dynamic isomorphism between subject and object—that is, the growing alignment between subjective ideas and objective reality. Yet, this margin of error, far from being purely technical, is itself a political discourse within science. In *Impossible Probability*, error is classified into two fundamental types:

1. The probability of error in sample representativeness. This is measured by the *inversion of the sample*:
    - 1/N in universes of infinite subjects or options.
    - 1/∑xi in universes of limited options, where ∑xi represents the sum of direct scores or frequencies.
2. Critical Reason (Razón Crítica). This is expressed as a critical probability p(xc) equal to a percentage X (error or reliability) divided by 100, applied to an ideal or maximal trend. Critical Reason, explained in Section 11 of *Introduction to Impossible Probability*, provides the rational threshold by which political-scientific policies determine whether a given deviation is acceptable as "truth" or must be rejected.

Thus, scientific truth is always contingent: it lives in the space between empirical representativeness and the rational margins of error set by the political-scientific community.

10.3. Science as Political Praxis

Science is political not only because every definition of truth is, at bottom, a moral definition of reality—what ought to be the moral truth, the universal moral, or the "real moral universe." Science is also political in a more direct sense: it is an abstract entity that depends upon the *volition*, purpose, and strategic objectives pursued by scientific policy through its research projects.

Indeed, the very definition of reality that science adopts is essentially a dependent variable of the *interests of the prevailing political-scientific framework*. These interests

are identified and codified through the political ideology that orients scientific policy toward particular goals or ideals. In this sense, scientific activity cannot be disentangled from praxis—it is a practice shaped by ideological commitments.

In *Impossible Probability*, this political dimension of science is not ignored but explicitly integrated into its statistical logic. The political definition of ideals determines the nature of sample distributions and thus conditions whether dispersion will be normal, maximal, or omega-type. By framing scientific reasoning as a dialogue between empirical data, rational critique, and political ideals, *Impossible Probability* insists that the production of *knowledge* is never neutral but always situated in a *moral and political horizon*.

10.4. Praxis in Impossible Probability

In *Impossible Probability*, ***science transforms itself*** *into praxis*—that is, practical reason—through two fundamental pathways. First, through the explicit acceptance by scientific policy of *margins of error*, namely: (a) the error of sample representativeness, and (b) the critical reason (*razón crítica*), which is the rational threshold applied to ideal tendencies. Second, through the deliberate determination of the *ideological objectives* that guide scientific policy.

By integrating these two pathways, *Impossible Probability* situates statistical reasoning at the intersection of empirical rigor and ideological orientation. Science is thus not simply a set of abstract calculations but an active political practice: *the rational acceptance of error margins and the definition of goals that shape how reality should ideally be understood and transformed*.

10.5. Ideological Objectives and Sample Distribution

The ideological objectives—or ideals—of scientific policy are what ultimately determine the form of sample distribution. Depending on the political and scientific ideals at stake, the *ideal distribution* of a given sample will take on different

configurations. This means that every scientific study is not only an empirical exercise grounded in facts but also a *political act* that reflects *ideological praxis*.

Put differently, science is *positive* in that it is based on observable phenomena, but it is not *neutral*. The structure of the sample, and the statistical conclusions that follow from it, are guided by the ideological commitments that shape what counts as truth, what counts as error, and what objectives should be pursued.

As a result, the "ideal" sample distribution is not a purely statistical abstraction but a reflection of the political ideals guiding the scientific project.

10.6. Models of Sample Distribution in Impossible Probability

Within the framework of *Impossible Probability*, at least two fundamental models of *ideal sample distribution* can be distinguished:

1. **Normal Sample Distribution (Normal Statistical Models).** These occur when sample dispersion oscillates between zero and maximum, except in the case where the object of study is a *sample of zeros*.
2. **Omega Sample Distribution (Omega Models).** These occur when the number of *ideals* within the sample is greater than one but less than N; in other words, when the number of politically or scientifically defined ideals is between two and N−1.

This classification reflects the recognition that dispersion in statistical models is always conditioned by the number of ideals established by scientific policy. A system with a single ideal tends toward maximum dispersion, while one with multiple competing ideals (but not encompassing the whole) tends toward *omega-type* distributions.

10.7. Definition of Normal Dispersion and the Sample of Zeros

In *Impossible Probability*, a dispersion is said to be *normal* when it oscillates between zero and maximum. This only occurs under two conditions:

1. Dispersion tends toward zero because the political-scientific ideal is equality of opportunity across all subjects or options.

2. Dispersion tends toward maximum because, out of all N subjects or options, there is only one predefined *ideal* subject or option that must achieve the *Maximum Empirical Probability Possible* (p(xi)=1. In this case, the remaining N−1 subjects or options must tend toward the *Minimum Empirical Probability Possible* (p(xi)=0).

Thus, models of *normal distribution* are those in which dispersion ranges from zero to maximum—*except* in the case of *samples of zeros*.

A **sample of zeros** occurs when the political-scientific ideal dictates that the empirical probability of every subject or option must be zero, as in the elimination of negative qualities or phenomena. In such cases, every subject or option in N tends toward an empirical probability of zero. The absolute value of every Bias Level is therefore equal to the inversion of N (1/N). The *Total Bias* (the sum of all absolute biases), being equal to N·1/N=1, so the arithmetic mean is 1/N.

Hence, for a sample of zeros, in the Second Method of *Impossible Probability*—unlike in traditional statistics—dispersion can only equal 1/N. This makes the *sample of zeros* a unique case where the rules of dispersion behave differently, underscoring the specificity of the Second Method compared to classical approaches.

10.8. Definition of Omega Models

In *Impossible Probability*, a model is defined as an **omega model** when, within a set N, there exists a number of *ideal* subjects or options greater than one but strictly less than N. These *ideal* elements are considered such not by chance, but because they are **politically defined as ideals** by the guiding framework of scientific policy.

Thus, omega models differ from normal models in that their structure is inherently plural: rather than a single option being elevated as the sole ideal (as in normal maximum-dispersion cases), omega models acknowledge the coexistence of multiple politically valued ideals. This creates a specific mathematical dynamic where

dispersion no longer oscillates simply between zero and maximum, but instead stabilizes around an **ideal dispersion** determined by the distribution of the subset of ideals, denoted as Ω=Omega

10.9. Example of a Normal Model: Multicultural Attitudes

Consider, within an *experimental design*, a study conducted to measure **progress in multicultural attitudes** within a given population. Suppose that this population has recently undergone a *public-awareness* campaign *aimed at reducing racism* and promoting *social integration*. After the campaign, researchers administer a sufficiently representative **attitude test** to evaluate whether *levels* of racial integration have increased.

Within this test, *imagine a specific item* asking respondents: *"Do you believe that racial integration improves social relations?"* The possible answers could be:

- "Yes, racial integration improves social relations."
- "No, racial integration does not improve social relations."
- "Don't know / No answer."

If scientific policy were to establish that only one of these options—namely, *"Yes, racial integration improves social relations"*—should be considered the political-scientific ideal, then the distribution of responses would be expected to converge toward this outcome. The ideal would be that this option tends toward the **Maximum Empirical Probability Possible** (p(xi)=1), while the other two options tend toward the **Minimum Empirical Probability Possible** (p(xi)=0).

In such a case, the model under consideration would be a **normal model of maximum dispersion**: one in which dispersion oscillates between zero (equality of opportunity) and maximum (the elevation of a single ideal option). This corresponds to the **Maximum Theoretical Mean or Standard Deviation Possible**, situating the model clearly within the category of *normal distribution models*.

10.10. Example of an Omega Model: Attitudes Toward an Accident

Now, let us consider a different test item in the same survey, where multiple responses may be chosen simultaneously:

“Suppose you are walking down the street and see a person of a different skin color who has had an accident. There is no one else around besides you and this injured person. What would your reaction be?”

The options provided might include:

- Pass by without helping.
- Pretend you did not see anything.
- Claim ignorance to avoid involvement.
- Ask the person what is wrong, if they can respond.
- Try to help them if it is within your means.
- Call an ambulance if necessary.

If the political-scientific ideal were to establish that, in such a situation, the *ideal responses* are exclusively:

- “Ask the person what is wrong.”
- “Try to help them if possible.”
- “Call an ambulance if necessary.”

—then the statistical model would no longer fall under the category of a normal model. Why*? Because the number of ideal responses is greater than one yet less than the total N*. This transforms the system into an **omega model**, where the set of politically defined ideals Ω=Omega (here, three options) coexists within the broader universe of possible responses.

In omega models, dispersion does not merely oscillate between zero and maximum, as in normal models. Instead, it tends toward an **omega-specific ideal dispersion**, one that reflects the simultaneous presence of multiple valued outcomes.

10.11. Differences Between Normal and Omega Models

The principal distinction between **normal models** and **omega models** lies in the typical behavior of sample dispersion. In normal models, dispersion tends to oscillate between two poles:

- **Zero dispersion**, corresponding to complete equality of opportunity across all subjects or options.
- **Maximum dispersion**, which arises when, within the set N, only a single subject or option is politically defined as ideal and therefore tends toward the **Maximum Empirical Probability Possible** (p(xi)=1), while all other N−1 subjects or options must tend toward the **Minimum Empirical Probability Possible** (p(xi)=0).

The only exception to this rule within normal models is the case of the **zero-sample model**, in which the political-scientific ideal stipulates that the empirical probability of every subject or option must be zero (i.e., the direct score or frequency of all negative qualities is eliminated). In this scenario, the dispersion takes on a unique form and is not governed by the same oscillatory behavior.

In contrast, omega models do not behave in this binary manner. In omega models, dispersion **neither tends to zero nor to maximum**, but instead stabilizes around an **ideal omega dispersion**. This is because more than one subject or option is simultaneously defined as ideal, yet the total number of ideals remains strictly less than N. Hence, the structure of dispersion is determined by a more complex equilibrium that balances multiple politically defined ideals against the non-ideal remainder.

10.12. Definition of the Omega Factor (Ω)

In *Impossible Probability*, the behavior of **ideal dispersion** is formalized through the concept of the **omega factor**, denoted as Ω=Omega. The omega factor represents the

number of subjects or options within N that are defined as **ideals** by the prevailing scientific policy.

Consider the following example item, already used in previous sections:

“If you are walking down the street and see a person of a different skin color who has had an accident, and there is no one else present but you and that person, what would your reaction be?”

The set of all possible responses is:

N={Pass by, Pretend not to notice, Refuse to get involved, Ask what is wrong,

Try to help, Call an ambulance}

Suppose the politically defined **ideals** are those responses oriented toward assistance:

$$\Omega = \{\text{Ask what is wrong},\ \text{Try to help},\ \text{Call an ambulance}\}$$

The remainder of the set, consisting of the **non-ideal responses**, is therefore:

$$N - \Omega = \{\text{Pass by},\ \text{Pretend not to notice},\ \text{Refuse to get involved}\}$$

By definition, the empirical probabilities of all **non-ideal responses** (N−Ω) must tend toward zero. Conversely, the probability mass is concentrated within Omega, Ω, with the ideal being that the total probability within Ω, Omega, equals unity.

$$(\sum_{\Omega} p(x_i) = 1)$$

Thus Ω, Omega, is the mathematical representation of the set of **politically idealized options**, and the difference N−Ω quantifies the number of non-ideal subjects or options whose empirical probability is normatively expected to vanish.

10.13. Political Dimension of the Definition of Ideals

It must be emphasized that any definition of what counts as **ideal** is inherently **political**. An ideal is not merely a descriptive property of reality; it is a **normative projection** of how reality ought to be structured according to a given political-scientific framework. In this sense, an **ideal definition of reality**—often aligned with utopian aspirations—is necessarily a political act, because it prescribes the transformation of empirical reality into an idealized order.

Within *Impossible Probability*, this insight is crucial: the process of defining ideals reflects the transition from **phenomenological objectivity** (what appears in experience) to a **subjectively posited ideal reality** that scientific policy seeks to make objective and real through empirical implementation. Mathematics, in this context, is conceived not only as a descriptive language but also as a tool for **idealization**—a method of formalizing the envisioned order so that it may be realized empirically.

In short, the omega framework illustrates that the designation of ideals is never neutral but deeply tied to political decisions about what should count as valuable, normative, or desirable in the *structuring of social, scientific, or experimental reality*.

10.14. Construction of an Ideal Omega Model

The formal construction of an **omega model** begins at the moment when, given a total set N, the subset of **ideal subjects or options** Ω, Omega, is explicitly defined. By definition Omega, Ω, must contain more than one and fewer than N elements, i.e., $2 \leq \Omega \leq N-1$, between 2 and N-1. Once the ideal subset Omeg, Ω, has been identified, the complement—consisting of all **non-ideal subjects or options**—is given by N−Ω.

It is only after this partition is established that one can begin constructing what is referred to in *Impossible Probability* as the **ideal model of reality**: a framework in which empirical distributions are aligned with the political-scientific definition of what constitutes the "ideal." The construction process involves imposing normative constraints: ideals must cluster around an empirically defined target probability, while non-ideals are constrained toward vanishing probabilities. This dual movement—allocation of probability mass to the ideals and withdrawal of probability from the non-ideals—generates a dispersion pattern that is neither zero nor maximum, but one that expresses the **ideal equilibrium of the omega model**.

10.15. The Ideal Bias Level in Omega Models

When the political-scientific ideal is fully realized, every subject or option within Omega, Ω, (the set of ideals) is assigned **equal importance**. This equality implies that all ideal subjects must share the same empirical probability. If, however, the ideals are not equally weighted—some being considered politically more important than others—then the appropriate method of adjustment is the **Effective Distribution** model, which is developed in Section 22 of *Impossible Probability*.

In the simpler case, where all ideals are equal in importance, the situation unfolds as follows:

- Every **non-ideal subject or option** (N−Ω) must tend toward an empirical probability of zero, since their nature is defined as politically non-ideal.
- The **sum of all probabilities across the set N** must equal one, as required by the axioms of probability.

From these two premises it follows necessarily that the **total probability must be concentrated in Omega, Ω**. Therefore, the **ideal probability** assigned to each member of Omega, Ω, is given by the reciprocal of the number of ideals:

$$p_{\text{ideal}} = \frac{1}{\Omega}$$

where:

- Ω= Omega = the number of politically ideal subjects or options within N, bounded between 2 and N−1.

This leads to an elegant result regarding the **Ideal Bias Level (Nivel de Sesgo Ideal)**. For every ideal subject or option, the Ideal Bias Level is equal to the difference between the **ideal probability** (the reciprocal of Ω, 1/Ω) and the **baseline probability under equality of opportunity** (the reciprocal of N, 1/N:

$$\text{Ideal Bias Level} = \frac{1}{\Omega} - \frac{1}{N}$$

This formula captures the essential departure of the omega model from the normal distribution framework. While in normal models the baseline for comparison is equality of opportunity across all N, in omega models the redistribution of probability among a smaller set of politically defined ideals generates a systematic deviation—an **idealized bias**—that reflects the normative structure of the model.

10.16. Biases of Non-Ideal Subjects or Options

If every element within the set of ideals Omega, Ω, has an **Ideal Bias Level**, then the **sum of all Ideal Bias Levels** must logically equal the product of the Ideal Bias Level and the number of ideals, Ω. In other words, the aggregate bias across all politically defined ideals is simply the uniform bias per ideal multiplied by their total count:

$$\sum(\text{Ideal Bias Levels}) = \left(\frac{1}{\Omega} - \frac{1}{N}\right) \cdot \Omega$$

At the same time, political-scientific definitions require that all **non-ideal subjects or options** (N−Ω) have a probability of zero. Since their empirical frequency is constrained to vanish, their **Bias Level** is necessarily the **Maximum Possible Negative Bias**, which is equal to the reciprocal of N:

$$\left|(p(x_i) = 0) - \frac{1}{N}\right| = \frac{1}{N}$$

Thus, for every non-ideal subject, the bias is fixed at the maximum negative deviation from the theoretical baseline of equality of opportunities. Extending this to the entire non-ideal subset, the **sum of all Bias Levels of non-ideals** is equal to the product of the reciprocal of N and the number of non-ideal elements (N−Ω):

$$\sum(\text{Bias Levels of Non-Ideals}) = \frac{1}{N} \cdot (N - \Omega)$$

This formulation ensures that both sides of the probability distribution—ideals and non-ideals—are symmetrically accounted for in the structure of omega models.

10.17. Formula for the Ideal Total Bias

In *Impossible Probability*, the **Total Bias (Sesgo Total)** of any model is defined as the sum of the absolute values of all individual bias levels present in the sample. When applied to the omega model, the Ideal Total Bias can be derived directly by combining the contribution of the ideal subset (Ω) with that of the non-ideal subset (N−Ω):

$$\text{Ideal Total Bias} = \left(\frac{1}{\Omega} - \frac{1}{N}\right) \cdot \Omega \; + \; \frac{1}{N} \cdot (N - \Omega)$$

This equation elegantly unifies the **positive biases** (arising from politically designated ideals) with the **negative biases** (arising from politically designated non-ideals). In this sense, the Ideal Total Bias quantifies the maximum possible departure of the omega model from a purely equal distribution, under the conditions established by the political-scientific framework.

10.18. The Ideal Mean Deviation

Once the Ideal Total Bias has been determined, the next step is to calculate the **Ideal Mean Deviation** (Desviación Media Ideal). By definition, the mean deviation is the arithmetic mean of the absolute bias values across the entire set of N subjects or options. Therefore, the Ideal Mean Deviation in an omega model is equal to the Ideal Total Bias divided by N:

$$\text{Ideal Mean Deviation} = \frac{\text{Ideal Total Bias}}{N}$$

Substituting the formula for the Ideal Total Bias yields:

$$\text{Ideal Mean Deviation} = \frac{\left(\frac{1}{\Omega} - \frac{1}{N}\right) \cdot \Omega \; + \; \frac{1}{N} \cdot (N - \Omega)}{N}$$

Equivalently, this can be understood as multiplying the Ideal Total Bias by the **baseline probability of chance** (1/N). This reflects the principle that the Mean Deviation in *Impossible Probability* is not an abstract construct but the **normalized average divergence from equality of opportunities**, incorporating both the positively privileged ideals and the negatively constrained non-ideals.

Thus, the Ideal Mean Deviation becomes a critical tool for diagnosing the **degree of structured inequality** embedded within an omega model. It highlights how much the empirical distribution, when aligned with political-scientific ideals, deviates from the neutral benchmark of uniform probability.

10.19. Ideal Variance in Omega Models

The **Ideal Variance** builds directly on the concept of the Ideal Mean Deviation. In *Impossible Probability*, the Mean Deviation measures the average absolute divergence of empirical probabilities from their theoretical benchmarks, while the Variance extends this by squaring those deviations before averaging them. This squaring operation magnifies larger deviations, allowing researchers to capture the intensity of dispersion, not just its average level.

In omega models, where political-scientific ideals determine that all ideal subjects or options (Ω) should converge toward the **ideal probability** 1/Ω, while all non-ideal elements (N−Ω) should converge toward probability zero, the variance can be derived systematically.

- **For the ideal subset (Ω)**: each element's deviation is given by 1/Ω−1N. Squaring this deviation and multiplying by the number of ideals (Ω) yields their total contribution to variance.
- **For the non-ideal subset (N−Ω)**: each element's deviation is given by −1N (Maximum Negative Bias). Squaring this deviation and multiplying by the number of non-ideal subjects (N−Ω) yields their total contribution.

Combining both parts and dividing by N (i.e., averaging across the whole universe), we obtain the **Ideal Variance**:

$$\text{Ideal Variance} = \frac{\left(\frac{1}{\Omega} - \frac{1}{N}\right)^2 \cdot \Omega \; + \; \left(\frac{1}{N}\right)^2 \cdot (N - \Omega)}{N}$$

This formula ensures that both ideal and non-ideal contributions to dispersion are rigorously balanced. It expresses, in compact form, the **expected squared deviation from theoretical equality of opportunities** when political-scientific ideals introduce structured differentiation within the model.

10.20. Ideal Standard Deviation

By definition, the **Standard Deviation** is the square root of the variance. It translates the abstract squared differences of the variance back into the same scale as the original probabilities, making it more interpretable in applied contexts.

Accordingly, in omega models, the **Ideal Standard Deviation** is simply the square root of the Ideal Variance:

$$\text{Ideal Standard Deviation} = \sqrt{\frac{\left(\frac{1}{\Omega} - \frac{1}{N}\right)^2 \cdot \Omega \; + \; \left(\frac{1}{N}\right)^2 \cdot (N - \Omega)}{N}}$$

This measure encapsulates the **average dispersion around the political-scientific equilibrium** established by the ideals. In practical terms, it provides a direct index of how far, on average, the observed structure deviates from the expected balance between ideals and non-ideals.

10.21. Importance of Omega Models in *Impossible Probability*

Omega models occupy a central position in *Impossible Probability* because they allow the rational critique of studies where **more than one political-scientific ideal** is embedded in the same sample. Unlike normal models, which only oscillate between zero dispersion (equality of opportunities) and maximum dispersion (a single dominant ideal), omega models introduce a structured middle ground: dispersion shaped by multiple, but not universal, ideals.

Through omega models, one can calculate the **ideal trends** that serve as benchmarks for critical analysis:

- **Ideal probability** for each ideal: $1/\Omega$
- **Ideal Bias Level**: $1\Omega - 1N$
- **Ideal Total Bias**: $(1\Omega - 1N)\cdot\Omega + 1N\cdot(N - \Omega)$
- **Ideal Mean Deviation**: $\{(1\Omega - 1N)\cdot\Omega + 1N\cdot(N - \Omega)\} : N$
- **Ideal Variance**: $\{(1\Omega - 1N)^2\cdot\Omega + (1N)^2\cdot(N - \Omega)\} : N$
- **Ideal Standard Deviation**: $\sqrt{\{\{(1\Omega - 1N)^2\cdot\Omega + (1N)^2\cdot(N - \Omega)\} : N\}}$

These quantities not only establish the **mathematical structure** of omega models but also provide the foundation for **critical rational tests**. Specifically, the scientific community can apply the concept of a **critical ratio** (*razón crítica*), multiplying the ideal trend by a percentage X of error or reliability, to evaluate whether the observed distribution in empirical data sufficiently approximates the politically defined ideal.

Thus, omega models are indispensable for bridging **mathematical formalism, political ideology, and empirical critique**. They formalize the link between probability structures and the moral-political dimensions of scientific practice, allowing probability theory to serve as both a descriptive and normative tool.

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